Edge Detection Optimization Using Fractional Order Calculus

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Abstract: *In computer vision and image processing, time and quality are major factors taken into account. In edge detection process, the smoothing operation by a low-pass filter is commonly performed first in order to reduce noise effect. However, performing the smoothing operation partially requires additional computational time and alters true edges as well. Attempting to resolve such problems, a new approach dealing with edge detection optimization is addressed in this paper. For this purpose, a short edge detector algorithm without smoothing operation is proposed and investigated. This algorithm is based on a fractional order mask used as kernel of convolution for edge enhancement. It has been shown that in the proposed algorithm, the smoothing pre-process is no longer necessary; because, the efficiency of our fractional order mask is expressed in term of immunity to noise and the capability of detecting edges. Simulation results show how the quality of edge detection can be enhanced on adjusting the fractional order parameter. Then, our proposed edge detection method can be very useful in real time applications in some fields such as, satellite and medical imaging.*

Keywords: *Edge detection, fractional order calculus, computational time, smoothing operation, performances evaluation.*

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1. Introduction

Edge detection, which is one of the most important issues of image processing, refers to the process of identifying and locating sharp discontinuities in an image. These discontinuities are the abrupt changes in the intensity or colour that may occur at the boundaries of objects. Edge detection is used in feature extraction, segmentation, and motion analysis. Many edge detection algorithms have been devised using a variety of approaches. For surveys on edge detectors, see for example [10, 16, 19, 28]. The main steps involved in edge detection operations are noise filtering, edge enhancement, and edge point detection using some detection rule such as thresholding.

The majority of edge detectors may be grouped into two categories: gradient-based edge detectors that search for edges as extrema of the first derivative of the image intensity, and Laplacian-based edge detectors where edges are detected as zero crossings in the second derivative of the image. Since the resulting operators are sensitive to noise, smoothing filters are applied to reduce high frequency noise. This requires designing an appropriate filter which will introduce additional computations. For high-performance hardware implementation, Field Programmable Gate Array (FPGA) device have been used for example in [1, 27] to efficiently implement Sobel operator-based edge detection.

Recently, more and more fractional order calculus-based techniques were developed and applied in the fields of signal and image processing. In fractional order calculus, the operations of differentiation and integration are generalized to non-integer order according to several definitions including Grünwald-Letnikov definition, Riemann-Liouville definition, and Caputo definition [4]. Applications in signal processing include resolution of overlapped peaks [17], and detection of Electrocardiogram (ECG) waves [6, 7, 8, 11]. In image processing, fractional order calculus-based masks were used in dealing with image texture enhancement [15, 24], image denoising [12, 13, 14], image segmentation [21, 22], and edge detection [5, 9, 18, 20, 23]. In edge detection process, the smoothing operation by a low-pass filter is commonly performed first in order to reduce noise effect. However, performing the smoothing operation partially, requires more computational time restricting real time applications, as well as affects the quality of edge detection on moving true edges from their actual locations.

Attempting to overcome smoothing problems, the contribution of this work is the design of a short edge detector algorithm, using a fractional order mask as kernel of convolution for edge enhancement. In term of robustness to noise of the fractional order calculus-based masks [5, 18], it has been shown that the smoothing process is no longer necessary in our approach. In the proposed algorithm, some criteria of Canny edge detection are applied [3]. This choice is justified by the fact that, Canny edge detector algorithm is the most optimal and widely used in image processing; because, it provides robust detection due to the three optimization criteria of good detection, good localization, and single response to an edge.

The rest of the paper is organized as follows: section 2 presents a background concerning the Grünwald-Letnikov definitions of left-handed and right-handed fractional derivatives. The design of the fractional order mask is described in section 3. Section 4 presents the contribution work. Some simulation results are reported and discussed in section 5. Finally, section 6 presents our conclusions.

2. Background

In original theory, the derivative of integer order n for a given function f of variable t, is a finite difference approximation of the form:

$$f^{(n)}(t) = \lim_{h\to 0^+} \frac{1}{h^n} \sum_{k=0}^{n} (-1)^k \binom{n}{k} f(t-kh) \tag{1}$$

Where $\binom{n}{k}$ are the binomial coefficients, and h is the differentiation step.This can be generalized to derivatives of fractional order α and leads to Grünwald-Letnikov left-handed and right-handed fractional derivatives. According to [26], the left-handed and right-handed derivatives in a domain $\alpha<t<b$ are defined by Equations (2) and (3) respectively.

$${}^{GL}_{a}D^{\alpha}_{t} f(t) = \lim_{h\to 0^+} \frac{1}{h^\alpha} \sum_{k=0}^{\infty} (-1)^k \binom{\alpha}{k} f(t-kh) \tag{2}$$

$${}^{GL}_{t}D^{\alpha}_{b} f(t) = \lim_{h\to 0^+} \frac{1}{h^\alpha} \sum_{k=0}^{\infty} (-1)^k \binom{\alpha}{k} f(t+kh) \tag{3}$$

Where $\binom{\alpha}{k} = \frac{(\alpha)(\alpha-1)\dots(\alpha-k+1)}{k!}$ is the generalization of the binomial coefficients. The preceding relations can be applied to image processing with some considerations: First, we agree that in image processing the digitalizing step is often taken as h=1. Second, consider a finite interval centred in t, for a given (N+1) finite number of samples such as N=$\lfloor (t-a)/h \rfloor = \lfloor (b-t)/h \rfloor$), where$\lfloor\,.\,\rfloor$ designates the integer part. Then, relations (2) and (3) can be approximated respectively by:

$${}^{GL}_{a}D^{\alpha}_{t} f(t) \cong \sum_{k=0}^{N} \omega_k . f(t-k) \tag{4}$$

$${}^{GL}_{t}D^{\alpha}_{b} f(t) \cong \sum_{k=0}^{N} \omega_k . f(t+k) \tag{5}$$

Such as

$$\omega_k = \begin{cases} 1, & k=0 \\ (-1)^k \binom{\alpha}{k}, & k = 1.2.3\ \dots \end{cases} \tag{6}$$

3. Design of the Fractional Order Mask

In this section, we are focusing on how the fractional order mask can be obtained. At start, let to perform the difference between Grünwald-Letnikov left-handed and right-handed fractional derivatives given by (4) and (5) respectively.

$$\begin{aligned} {}^{GL}_{a}D^{\alpha}_{t} f(t) - {}^{GL}_{t}D^{\alpha}_{b} f(t) \cong \\ \omega_1 f(t-1) + \omega_2 f(t-2) + \dots + \omega_N f(t-N) \\ - \omega_1 f(t+1) - \omega_2 f(t+2) - \dots - \omega_N f(t+N) \end{aligned} \tag{7}$$

Yields

$${}^{GL}_{a}D^{\alpha}_{t} f(t) - {}^{GL}_{t}D^{\alpha}_{b} f(t) \cong \sum_{-N}^{N} c_k . f(t-k) \tag{8}$$

Where c_k are the coefficients of a 1D finite impulse response filter such as: c_0=0, $c_k=\omega_k$, and $c_{-k} = -\omega_k$. Note that, depending on whether the fractional order α is positive or negative; then, expression (8) can be a fractional order differentiation or integration.

Now, given an image $I(i,j)$ where i is the pixel position in x direction, and j is the pixel position in y direction. Let us denote by G_x the derivative in x direction, and by G_y the derivative in y direction. Then, relation (8) can be extended to 2D case as follows:

$$G_x = \sum_{k=-N}^{N} \sum_{l=-N}^{N} h_x(k,l) . I(i-k, j-l) \tag{9}$$

$$G_y = \sum_{k=-N}^{N} \sum_{l=-N}^{N} h_y(l,k) . I(i-l, j-k) \tag{10}$$

Where $h_x(k,l)$ is the kth and the lth coefficient in the horizontal mask referred to by M_x, and $h_y(l,k)$ is the lth and the kth coefficient in the vertical mask referred to by M_y such as $h_x(k,l)= h_y(l,k)=c_k$.

It follows that, $M_x = M_y^t$ and both of size $(2N+1)$ $(2N+1)$. The components M_x and M_y of the fractional order mask are represented in Tables 1 and 2 respectively.

Table 1. Operator M_x.

ω_N	...	ω_1	0	$-\omega_1$	...	$-\omega_N$
ω_N	...	ω_1	0	$-\omega_1$	...	$-\omega_N$
ω_N	...	ω_1	0	$-\omega_1$	...	$-\omega_N$
ω_N	...	ω_1	0	$-\omega_1$	...	$-\omega_N$
ω_N	...	ω_1	0	$-\omega_1$	...	$-\omega_N$
ω_N	...	ω_1	0	$-\omega_1$	...	$-\omega_N$
ω_N	...	ω_1	0	$-\omega_1$	...	$-\omega_N$

Table 2. Operator M_y.

ω_N	ω_N	ω_N	ω_N	ω_N	ω_N	ω_N
...	...	...	...	...	...	...
ω_1	ω_1	ω_1	ω_1	ω_1	ω_1	ω_1
0	0	0	0	0	0	0
$-\omega_1$	$-\omega_1$	$-\omega_1$	$-\omega_1$	$-\omega_1$	$-\omega_1$	$-\omega_1$
...	...	...	...	...	...	...
$-\omega_N$	$-\omega_N$	$-\omega_N$	$-\omega_N$	$-\omega_N$	$-\omega_N$	$-\omega_N$

4. Contribution Work

Concerning our contribution work, Table 3 below is assumed to be more explicit scheme. On the left, we presented a commonly used edge detector algorithm with a smoothing step, which will be termed the original algorithm. On the right, we presented the same edge detector algorithm without the smoothing step, which will betermed the modified algorithm.

Table 3. Original and modified algorithms.

Original algorithm	Modified algorithm
Smoothing	x (no operation)
Gradient determination	Fractional gradient determination
Non-Maxima suppression	Non-Maxima suppression
Thresholding	Thresholding

4.1. Comparison Aspects

The original edge detection method is characterized by the smoothing operation using a Gaussian filter, and gradient determination is achieved using a classical mask (e.g., Sobel mask) as kernel of convolution. In the counterpart, the proposed edge detection method is characterized by the absence of the smoothing operation and the fractional gradient determination is achieved using the fractional order mask described in the preceding as kernel of convolution. However, operations corresponding to non-maxima suppression and thresholding are shared between methods.

4.2. Proposed Modified Algorithm

- *Step*1: Fractional gradient determination: Most edge detection methods work on the assumption that an edge occurs where there is a discontinuity in the intensity function. The components G_x and G_y of the fractional gradient are computed by convolving the image with the corresponding operators M_x and M_y of the fractional order mask. Then, the magnitude and direction of the fractional gradient are computed using Equations (11) and (12) respectively.

$$|G| \cong |G_x| + |G_y| \tag{11}$$

$$\theta = \arctan(G_y/G_x) \tag{12}$$

- *Step* 2: Non-Maxima suppression: This operation is proposed by Canny [3] and aims to produce thin edges on removing non-maxima pixels along the gradient direction preserving the connectivity of contours. The gradient magnitude of the current pixel is compared with the gradient magnitude of two neighbours on either side lying along the gradient direction. If the gradient magnitude in the current position is greater than those on either side, it is declared a possible edge. If not, it is declared as background and so on for all pixels in the gradient.
- *Step* 3: Thresholding Canny defines lower and higher thresholds. So, the pixels lying above the lower threshold are edge candidates and the others are rejected. All pixels lying above the higher threshold are called seed regions for good edges. The idea is that all pixels above the lower threshold but connected to such a seed pixel are kept, whereas pixels above the lower threshold but not connected to such a seed are rejected. Thus, double thresholding avoid broken contours and efficiently remove small edge response due to noise.

5. Simulation and Discussion

For simulation purposes, on the one hand, in order to meet Canny method, we implemented the original algorithm using a Gaussian smoothing filter of variance σ=1.4 and Sobel operator for gradient determination. On the other hand, we implemented our modified edge detector algorithm using a 5x5 mask. The source code is written in Visual C# SDK 2010. The simulation was running on a Desktop equipped by a Dual Central Processing Unit (CPU) 1.8 MHz Intel Pentium (R) and 2GB of RAM. The running time is measured programmatically by instantiating the system timer. To keep the immunity to noise more effective, the mask should act as an integral operator. For this reason, the fractional order α is being adjusted inside the range $-1<a<0$. To achieve double thresholding, we notice good results are obtained when choosing 25 as lower threshold and 50 as higher threshold.

5.1. Fractional Order Optimization

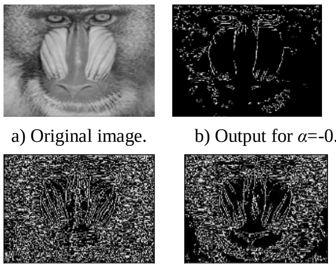
In this sub-section, the principle of application of the modified edge detector algorithm is targeted. Figure 1 below, illustrates an overview on how contours of test image 'Mandrill' (shown in Figure 1-a) are segmented progressively using different values of the fractional order parameter α.

In a first time, simulation is carried on choosing two arbitrary values of the fractional order parameter, which can be considered as low and high level of edge segmentation. As an example, edge map shown in Figure 1-b obtained at low level (α=-0.1), presents a lack of edge details. In the counterpart, edge map shown in Figure 1-c obtained at high level (α=-0.5), presents excess of edge details. The preceding results can be interpreted by the fact that, coefficients of the fractional order mask used as operator of convolution are expressed in function of the fractional order parameter α, which being adjusted leads to a variable amount of convolution magnitude. Concluding that, more is the decrease in absolute value of the fractional order parameter more is the decrease in convolution magnitude, and consequently more edge details are lost progressively, and vice versa. In fact, more faint edges details are interpreted commonly by noisy edges, however, a meaningful interpretation can be given is that, unless those due to residual noise, they are faint edges details less significant to human eye perception. This fact is in connection with the capability of the fractional order mask to reveal details progressively due to the adjustability of the fractional order parameter α. Contrarily, the other classical masks (e.g., Prewitt, Sobel) have constant coefficients leading to a constant amount of convolution magnitude, the reason why the property of revealing details progressively is restricted in this case.

In a second time, we will demonstrate how to retrieve the value of the fractional order parameter α, which allows to get desirable results. In this case, the optimal value of the fractional order parameter is iteratively searched. In fact, the optimal value of a is that for which, the performance of the detector is more improved. The theoretic tool we used to retrieve the optimal value more accurately is searching the value which maximizes the Pearson's correlation coefficient [25]. The Pearson's correlation coefficient defined in Equation (13) performs the measure of similarity between an edge image I_r considered as reference, and an edge image I_α depending on the parameter α.

$$r = \frac{\sum_i (x_i - x_m)(y_i - y_m)}{\sqrt{\sum_i (x_i - x_m)^2}\sqrt{\sum_i (y_i - y_m)^2}} \tag{13}$$

Where x_i and y_i are the intensity of the ith pixel in I_r and I_a respectively, x_m and y_m are the mean intensity in I_r and I_a respectively. The plot of the correlation r in function of the fractional order parameter α is depicted in Figure 2. Consequently, the final edge map obtained is shown in Figure 1-d, where the correlation is maximized (r=0.687) for an optimal value α =-0.239. The dependency of the optimal value to image texture is noticed in several experiments.



a) Original image. b) Output for α=-0.1.

c) Output for α=-0.5d). d) Output for α=-0.239.

Figure 1. Results of the modified algorithm on varying a.

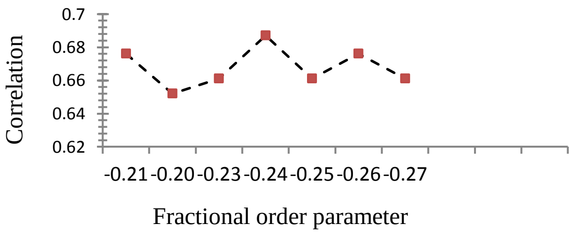
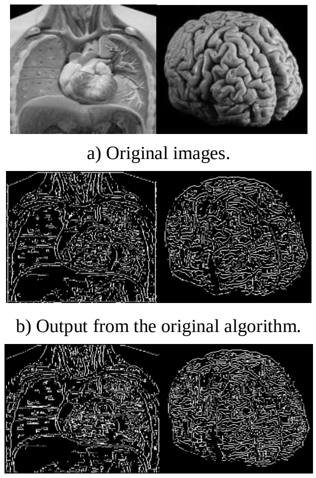


Figure 2. Plot of Pearson's correlation variation.

5.2. Comparing Results

In this sub-section, the comparison between the original and the modified algorithm is targeted. For this purpose, two test images ('Chest anatomy' and 'Brain') shown in Figure 3-a are tested. The output edge maps from the original algorithm are shown in Figure 3-b, while those from the modified algorithm are shown in Figure 3-c. We can see that edge maps are closer and it is difficult to distinguish the difference by human visual system. Here, the final edge maps of test images 'Chest anatomy' and 'Brain' are obtained for the optimal values α=-0.229 and α=-0.219 respectively.



a) Original images.

b) Output from the original algorithm.

c) Output from the modified algorithm.

Figure 3. Simulation results of original and modified algorithms.

5.3. Robustness to Noise

In order to illustrate the performances of our fractional order mask in term of robustness to noise, an additive zero mean Gaussian noise with 0.01 variance is used to obtain the noisy image 'Peppers' shown in Figure 4-a. So, the resulting edge map from the original algorithm is shown in Figure 4-b, and the resulting edge map from the modified algorithm corresponding to an optimal value α=-0.249 is shown in Figure 4-c. Visually, edge maps seem to be closer unless few extra edge points due to residual noise in the case of our proposed method.

a) Original image. b) Original algorithm. c) Modified algorithm.

Figure 4. Test of a noisy image using both algorithms.

5.4. Computational Time Reduction

Reducing the computational cost in the original edge detection method is one of the main problems addressed in this paper. This can be accomplished thanks to the following considerations: On the one hand, there is no time consumed in the smoothing operation; because, in the case of our edge detection method this step is skipped. On the other hand, the computational complexity of the convolution is simplified exploiting the symmetry of the fractional order mask. As an example, consider the 5x5 image block *I(x,y)* labelled z_1 to z_{25} beginning from left to right and from top to down. The convolution with the horizontal and the vertical masks is performed according to the following Equations given by (14) and (15) respectively. Note that, each convolution involves only 2 multiplications and 19 additions.

$$M_x * I = \omega_2\left(\sum_{i=0}^{4} z_{5i+1} - \sum_{i=1}^{5} z_{5i}\right) + \omega_1\left(\sum_{i=0}^{4} z_{5i+2} - \sum_{i=1}^{5} z_{5i-1}\right) \quad (14)$$

$$M_y * I = \omega_2\left(\sum_{i=1}^{5} z_i - \sum_{i=21}^{25} z_i\right) + \omega_1\left(\sum_{i=6}^{10} z_i - \sum_{i=16}^{20} z_i\right) \quad (15)$$

In fact, the optimization process of the fractional order is supported programmatically. The running time measurement is accomplished via several tests using various images. Then, the running time of our modified algorithm is averaging 65% of the running time in the original edge detection algorithm.

5.5. Performance Evaluation

The performance assessment can be made subjectively by visual analysis of the detected edge map, or objectively comparing the edge map obtained by an edge detector with its ground truth. For this purpose, Pratt's figure of merit defined in [2] and given by Equation (16) is considered.

$$IMP = \frac{1}{\max(N_I, N_B),} \sum_{i=1}^{N_B} \frac{1}{1+ad_i^2} \quad (16)$$

Where N_I, N_B are the points of edges in the image and ground truth image respectively, d_i is the distance between an edge pixel and the nearest edge pixel of the ground truth and a=1/9 is an empirical calibration constant was used. Test image and its ground truth (edge map) extracted from Berkeley Segmentation Data Set and Benchmarks 500 (BSDSB500) are shown in Figure 5. Then, the output of our edge detector for an optimal α=-0.25 leads to IMP=0.668, while the output of the original edge detector leads to IMP=0.559. This difference is a result due to the additional selectivity control made by the fractional order parameter.

Figure 5. Test image and its ground truth edge image.

6. Conclusions and Future Work

In this paper, we addressed the possibility to avoid the smoothing disadvantages. For this purpose, we proposed a modified algorithm, where the smoothing operation by low-pass filter is no longer necessary. Results of simulation show that it is usable when applying a fractional calculus-based mask as kernel of convolution instead of classical ones. Through this work, it has been shown that the computational cost can be reduced on skipping the smoothing step and exploiting the symmetry of our mask. To prove the robustness to noise of our mask, we used noisy test images. The optimization of the fractional order parameter is achieved programmatically using Pearson's correlation coefficient as a theoretic tool. The quality assessment of our modified algorithm is achieved using Pratt's figure of merit. The performances of our fractional order mask are expressed in term of robustness to noise, edge selectivity, and computational complexity reduction. Then, the advantage of our contribution can be very useful in real time applications. Finally, this study is a step forward towards a new approach of edge detection without primary smoothing operation thanks to fractional calculus. As long as this paper has demonstrated the potential of reducing time execution that is a challenging problem in real time applications. Then, our ultimate interests are extending the scope of this paper in such a way to bring future enhance of the work from performances point of view The improvement will include immunity to noise, masque size effects and feature extraction selectivity.

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