Edge Preserving Image Segmentation using Spatially Constrained EM Algorithm

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Abstract: *In this paper, a new method for edge preserving image segmentation based on the Gaussian Mixture Model (GMM) is presented. The standard GMM considers each pixel as independent and does not incorporate the spatial relationship among the neighboring pixels. Hence segmentation is highly sensitive to noise. Traditional smoothing filters average the noise, but fail to preserve the edges. In the proposed method, a bilateral filter which employs two filters - domain filter and range filter, is applied to the image for edge preserving smoothing. Secondly, in the Expectation Maximization algorithm used to estimate the parameters of GMM, the posterior probability is weighted with the Gaussian kernel to incorporate the spatial relationship among the neighboring pixels. Thirdly, as an outcome of the proposed method, edge detection is also done on images with noise. Experimental results obtained by applying the proposed method on synthetic images and simulated brain images demonstrate the improved robustness and effectiveness of the method.*

Keywords: *Gaussian mixture model, expectation maximization, bilateral filter, image segmentation.*

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1. Introduction

Image segmentation is the process of partitioning an image into non overlapping regions such that pixels within a region share some common characteristic such as intensity, color or texture. Various Segmentation Methods like thresholding, Region Growing, Edge Detection Techniques, Histogram based Methods, Clustering, Classifiers, Artificial Neural Networks, Model Based Techniques and Deformable models have been proposed. Segmentation finds wide range of application fields which include machine vision, medical imagery, object detection and recognition tasks.

The Gaussian Mixture Model (GMM) [2, 5] is an efficient method for classification problems since it has the capability to use prior knowledge to model the uncertainty in a probabilistic manner. Also, it requires minimum parameters-mean, variance and mixing coefficient for learning and these parameters can be efficiently estimated by the Expectation Maximization (EM) algorithm. While considering the drawbacks of GMM, the distribution does not depend on the pixel index and the spatial relationship between the neighboring pixels. Therefore segmentation is extremely sensitive to noise and illumination.

Mixture models based on Markov Random Field (MRF) [3, 6] are also in research focus for image segmentation. Zhang *et al*. [16] proposed methods of incorporating weighted arithmetic mean template and a weighted geometric mean template in the prior probability to make the GMM more robust to noise. A method of incorporating spatial information in prior probability distribution is proposed [11, 14]. Nguyen *et al*. [10] proposed the method of assigning varying weights to different pixels appearing in the window and used student's-t distribution which is more robust to noise than Gaussian distribution. Kalti *et al*. [7] proposed the method which calculates the spatial weight depending on intrinsic properties of the pixel and the neighborhood of the pixel.

The problem associated with traditional mean filter is that the edge is smoothened in addition to that the noise is averaged out. Bilateral filter is a popular edge preserving filter which is used for image denoising applications [1, 8, 15, 17, 18]. Bilateral filter is a combination of domain filter and range filter. Domain filtering averages image values with weights that fall off with distance. Range filtering averages image values with weights that decay with dissimilarity.

Based on the above considerations, in this paper, a new method is proposed for edge preserving segmentation of images with noise. Firstly, bilateral filter is applied to the image for edge preserving smoothing. In this step, noise variance is estimated from the image using the robust median estimator.

Secondly, the posterior probability is weighted with the Gaussian kernel in which, the weights decrease with distance from the neighborhood center. Thirdly, as an outcome of the proposed method, edge detection of the noisy image is also done in addition to segmentation of the image [9].

The remainder of the paper is organized as follows: Gaussian Mixture Model is detailed in section 2, bilateral filter is explained in section 3, the details of the proposed method are explained in section 4, the experimental results are given in section 5 and conclusions are given in section 6.

2. Gaussian Mixture Model

If K is the number of classes, the Gaussian mixture model assumes that each pixel is composed by K component densities mixed together with K mixing coefficients. The parameters are estimated by Maximum Likelihood (ML), and EM algorithm is used as an optimization method.

The Gaussian distribution can be written in the form

$$N(x/\mu,\sigma^2)=\frac{1}{(2\pi\sigma^2)^{1/2}}exp\left\{-\frac{1}{2\sigma^2}(x-\mu)^2\right\} \quad (1)$$

Where μ is the mean and σ^2 is the variance.
A mixture of Gaussians can be represented by:

$$p(x)=\sum_{k=1}^{K}\pi_k N(x/\mu_k,\sigma_k^2) \quad (2)$$

Each Gaussian density N(x/μ_k, $\sigma_k^{2)}$ is called a component of the mixture and has its own mean μ_k and variance σ_k^2. The parameters π_k are called the mixing coefficients, $\sum_{k=1}^{K}\pi_k=1$ and $0\leq \pi_k \leq 1$, From Baye's theorem, the posterior probability is given by:

$$\gamma_k(x)=\frac{\pi_k N(x/\mu_k,\sigma_k^2)}{\sum_{j=1}^{K}\pi_j N(x/\mu_j,\sigma_j^2)} \quad (3)$$

The log of the likelihood function is given by:

$$ln\, p(X/\mu,\sigma^2,\pi)=\sum_{n=1}^{N} ln\left\{\sum_{k=1}^{K}\pi_k N(x_n/\mu_k,\sigma_k^2)\right\} \quad (4)$$

Where $X=\{x_1, x_2, \ldots x_n\}$.

For maximizing the log likelihood, the derivatives of Equation (4) with respect to μ_k, σ_k^2 and π_k are set to zero.

$$N_k=\sum_{n=1}^{N}\gamma_k(x) \quad (5)$$

The parameters are obtained as:

$$\mu_k=\frac{1}{N_k}\sum_{n=1}^{N}\gamma_k(x)x_n \quad (6)$$

$$\sigma_k^2=\frac{1}{N_k}\sum_{n=1}^{N}\gamma_k(x)(x_n-\mu_k)^2 \quad (7)$$

$$\pi_k=\frac{N_k}{N} \quad (8)$$

The EM algorithm for GMM is explained in the following steps:

- *Step 1.* Initialize the parameters-means μ_k, variances σ_k^2 and mixing coefficients π_k and evaluate the initial log likelihood using Equation (4).
- *Step 2.* E Step. Evaluate the posterior probabilities using Equation (3).
- *Step 3.* M Step. Re-estimate the parameters using current posterior probabilities using Equations 5, 6, 7 and 8.
- *Step 4.* Evaluate the likelihood and check for convergence. If the convergence criterion is not satisfied, return to step 2.

The drawback of Gaussian Mixture Model is that it considers each pixel as independent and classifies it accordingly. It does not take into account the spatial correlation between the neighboring pixels. Hence the method is sensitive to noise and illumination.

3. Bilateral Filtering

Bilateral Filtering is a non-iterative and simple method for smoothing edges while preserving edges. It employs two filters–domain filter and range filter.

Gaussian low-pass filter is the domain filter and it computes a weighted average of pixel values in the neighborhood, in which, the weights decrease with distance from the neighborhood center. Range filter averages image values with weights that decay with dissimilarity. In Gaussian filters, weight of the pixels considered by distance from the axis of the filter is given by:

$$G_\sigma(x,y)=\frac{1}{2\pi\sigma^2}e^{-\frac{(x^2+y^2)}{2\sigma^2}} \quad (9)$$

Mathematically, the bilateral filter output at a pixel location p is given by:

$$I_F(p)=\frac{1}{W}\sum_{q\in s}G_{\sigma_s}(\|p-q\|)G_{\sigma_r}(|I(p)-I(q)|)I(q) \quad (10)$$

Where,

$$G_{\sigma_s}(\|p-q\|)=e^{-\frac{\|p-q\|^2}{2\sigma_s^2}} \quad (11)$$

is the geometric closeness function and

$$G_{\sigma_r}(|I(p)-I(q)|)=e^{-\frac{|I(p)-I(q)|^2}{2\sigma_r^2}} \quad (12)$$

is the gray level similarity function.

$$W=\sum_{q\in s}G_{\sigma_s}(\|p-q\|)G_{\sigma_r}(|I(p)-I(q)|) \quad (13)$$

is the normalization constant. $\|p-q\|$is the Euclidean distance between p and q and S is the spatial neighborhood of p.

The two parameters σ_s, the geometric spread in the domain and σ_r, the photometric spread in the image range control the behavior of the bilateral filter. A good range of σ_s value is [1.5 2.1] and the optimal value of σ_r changes as the noise standard deviation σ_n changes.

The noise variance is estimated from sub band HH_1 of the wavelet decomposition of the image by the robust median estimator [4] given by:

$$\hat{\sigma}=\frac{Median\,(|Y_{i,j}|)}{0.6745}, \quad Y_{i,j}\in\{HH_1\} \quad (14)$$

4. Proposed Method

The GMM does not take into account the spatial correlation among the neighboring pixels. Hence the segmentation is sensitive to noise. If the pixels are averaged, segmentation accuracy is improved, but the edges are not preserved due to smoothing by the filters.

Hence a new method is proposed to incorporate the spatial correlation among the neighboring pixels and also to preserve the edges in the image. The steps in the proposed method are given below.

- *Step 1.* Apply bilateral filter Equation (10) to the noisy image and obtain the smoothened image with edges preserved. The value of σ_r is obtained by estimating the noise variance with the robust median estimator.
- *Step 2.* Apply modified EM method step as given in Figure 1.
- *Step 3.* Perform image segmentation from the obtained model.
- *Step 4.* Fix upper threshold and lower threshold for the posterior probabilities. (0.7 and 0.3 in the proposed method) and perform edge detection.

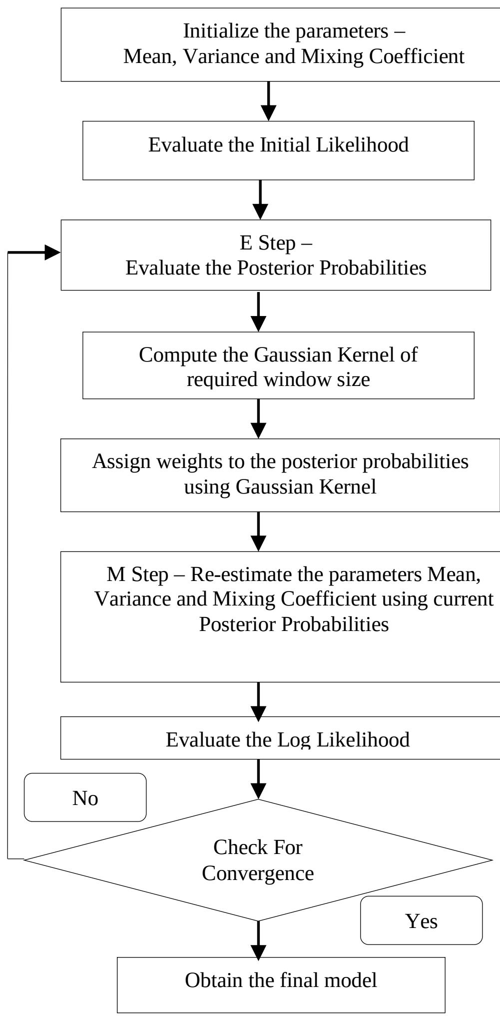


Figure 1. Proposed modified EM method.

5. Experimental Results

In this section, the performance of the algorithm on synthetic images as well as real data sets are presented. The system configuration used is Intel Core 2 Duo CPU @2. 53GHz with 1.98GB of Random Access Memory (RAM). The algorithm is carried out using MATLAB. The Misclassification Ratio (MCR) is employed to compare the effects obtained.

$$MCR = \frac{No.\ of\ \ misclassified\ Pixels}{Total\ number\ of\ \ Pixels} \qquad (15)$$

5.1. Synthetic Images

Similar synthetic images to those applied in [11, 12] are employed to test the effectiveness of the algorithm. The images are acquired at a resolution of 128x128. The image shown in Figure 2-a has 4 classes (K=4) with luminance values [0.25 0.5 0.75 1]. Gaussian noise with 0 mean and 0.005 variance is added to the image and shown in Figure 2-b.

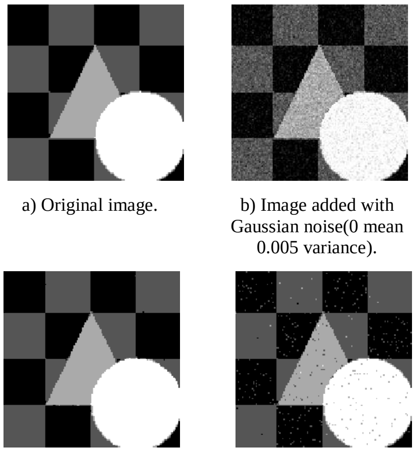
Standard GMM and proposed method are applied to the noisy synthetic image and the results obtained are shown in Figures 2-c and 2-d respectively. The stopping criteria for iterations, the minimum difference between two successive log likelihood is taken as 0.01. The number of iterations is 33 for the first experiment. It is inferred from Table.1 that the segmentation accuracy improves for the proposed method compared with the standard GMM and also with the method proposed in [11].

In the second experiment, for the same image noise with 0 mean and 0.03 variance is added and results are shown in Figure 3 As the noise increases the standard GMM fails to give acceptable segmentation accuracy.

For the third and fourth experiments, image with five classes (K=5) is taken with luminance values [0.2, 0.4, 0.6, 0.8 ,1] and the results shown in Figures 4 and 5 for a noise variance of 0.005 and 0.01 respectively.

For the fifth experiment, an image with four classes (K=4) is taken. Each square box in this image has the six of 64 x 64 pixels, and they have the same luminance values [0,1/3,2/3,1]. The results are shown in Figure 6 for a noise variance of 0.03.

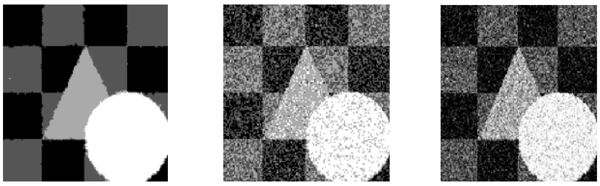
For the sixth experiment, an image with three classes (K=3) is taken. Each box in this image has a size of 32x 64 pixels and the 2048 pixels within each box have the same luminance value [1/3,2/3,1]. The results are shown in Figure 7 for a noise variance of 0.01. Tables 2 and 3 depict the performance comparison for third, fourth, fifth and sixth experiments.



a) Original image. b) Image added with Gaussian noise(0 mean 0.005 variance).

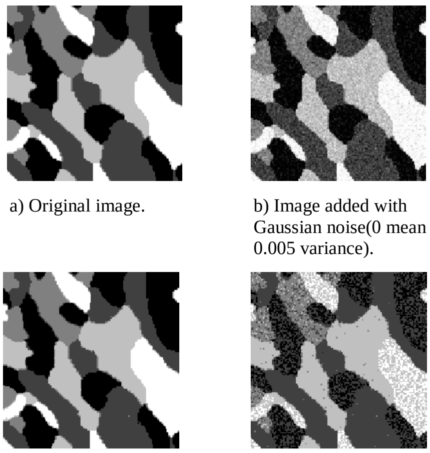
c) Standard GMM-EM (MCR=2.4%). d) Proposed method (MCR=0.27%).

Figure 2. First experiment (128 x128 image resolution).



a) Image added with Gaussian noise (0 mean 0.03 variance). b) Standard GMM-EM (MCR=38.15%. c) Proposed Method (MCR=1.22%).

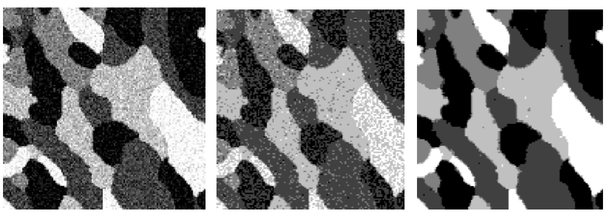
Figure 3. Second experiment (128x128 image resolution).



a) Original image. b) Image added with Gaussian noise(0 mean 0.005 variance).

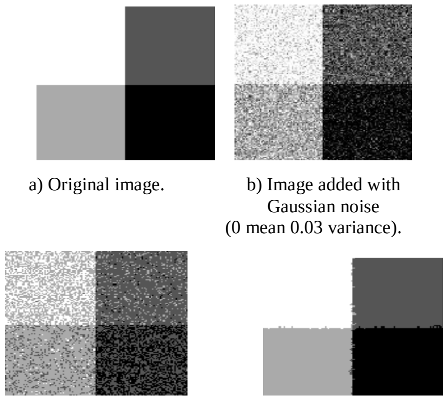
c) Standard GMM-EM (MCR=23.49%). d) Proposed method (MCR=0.71%).

Figure 4. Third experiment (128x128 image resolution).



a) Image added with Gaussian noise (0 mean 0.01 variance). b) Standard GMM-EM (MCR=28.26%. c) Proposed method (MCR=1.58%).

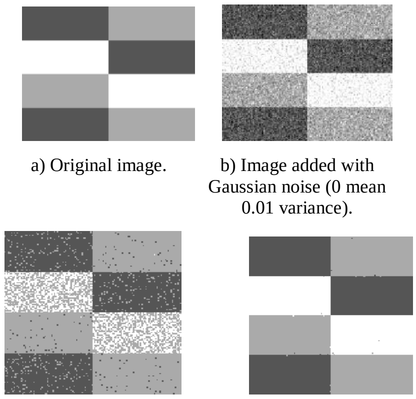
Figure 5. Fourth experiment (128x128 image resolution).



a) Original image. b) Image added with Gaussian noise (0 mean 0.03 variance).

c) Standard GMM-EM (MCR=34.68%). d) Proposed Method (MCR=0.32%).

Figure 6. Fifth experiment (128x128 image resolution).



a) Original image. b) Image added with Gaussian noise (0 mean 0.01 variance).

c) Standard GMM-EM (MCR=16.01%). d) Proposed method (MCR=0.13%).

Figure 7. Sixth experiment (128x128 image resolution).

Table 1. Performance comparison of the proposed method to other methods for the first and second experiments (Figures 2, and 3).

Methods	Gaussian Noise with Mean 0 and Variance			
	Variance=0.005		Variance=0.03	
	MCR	Time (secs)	MCR	Time (Sec)
Standard GMM	2.4%	0.8	28.15%	0.8
Method [11]	0.39%	-	-	-
Method [12]	-	-	1.13%	4.9
Proposed Method	0.27%	2.7	1.22%	3.0

Table 2. Performance comparison of the proposed method to other methods for the third and fourth experiments (Figures 4, and 5).

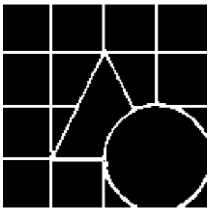
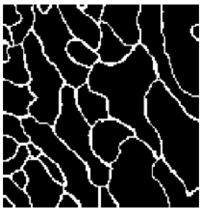
Methods	Gaussian Noise with Mean 0 and Variance			
	Variance=0.005		Variance=0.01	
	MCR	Time (secs)	MCR	Time (Sec)
Standard GMM	24.4%	0.8	28.09%	0.9
Method [11]	2.08%	-	-	-
Method [12]	-	-	0.73%	-
Proposed Method	0.65%	4.3	1.49%	4.4

Table 3. Performance comparison of the proposed method to other methods for the fifth and sixth experiments.

Methods	Gaussian Noise with Mean 0 and Variance			
	Variance=0.03 Experiment 5		Variance=0. 01 Experiment 6	
	MCR	Time (secs)	MCR	Time (Sec)
Standard GMM	30.12%	1.0	35.30%	0.8
Method [11]	0.21%	-	-	-
Method [12]	-	-	0.31%	13.4
Proposed Method	0.32%	2.6	0.13%	3.4

It is inferred from the experiments that the proposed method outperforms the standard GMM-EM and the method [11] with respect to both Misclassification Ratio (MCR) and computational time. Also, it is inferred that compared to the method [12] though MCR slightly increases, the computational time is reduced.

As a consequence of the proposed edge preserving EM method, edge detection is also performed on the noisy images and the output is shown in Figure 8.

a) Output for image used in experiment 1. b) Output for image used in experiment 3.


c) Output for image used in experiment 5. d) Output for image used in experiment 6.

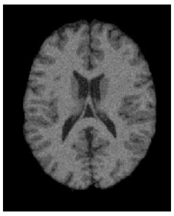
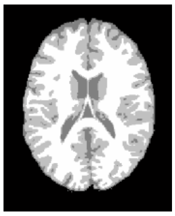
Figure 8. Edge detection output of the test images with noise variance 0.01.

5.2. Images from Brainweb

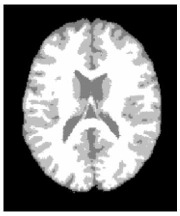
The effectiveness of the method is also tested on T1-weighted Magnetic Resonance (MR) brain images from the Brain web simulated Brainweb simulated brain database. The method is validated on simulated images with 40% inhomogeneity and 9% noise, 181x217x181 dimension 1x1x1 mm^3 spacing. The ground truth for the Brain Web dataset is the phantom atlas used to generate the simulated scans. The ground truth of the T1w images is known for comparisons. To evaluate the segmentation results, the segmentation of each class j is compared with the ground truth by using the Dice Similarity Index (DSI). [13] The *DSI S* (*j*) is defined as:

$$DSI = S(j) = \frac{2N_{p\cap g}(j)}{N_p(j) + N_g(j)} \quad (16)$$

Where $N_{p\cap g}(j)$ is the number of pixels classified as class j by both the proposed method and the ground truth. N_p (*j*) and N_g (*j*) represent the number of pixels classified as class j by the proposed method and by the ground truth, respectively. Figures 9 and 10 show the segmentation results of the simulated images from MRI brain phantom with 9% noise and 40% inhomogeneity for slices 90 and 120.

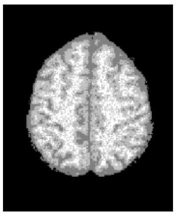
a) Original image. b) Ground truth image.

c) Standard GMM-EM. d) Proposed method.

Figure 9. Slice 90.


a) Original image. b) Ground truth image.




c) Standard GMM-EM. d) Proposed method.

Figure 10. Slice 120.

The brain images are segmented into three classes White matter, Gray matter and cerebro-spinal fluid. The Dice Similarity Index for the White Matter and Gray Matter are calculated for the standard GMM-EM method and the proposed method. It is inferred from Table 4 that significant improvement is achieved in the proposed method over standard GMM and also notable segmentation accuracy.

Table 4. Performance comparison of the proposed method to gmm-em method for brainweb T1 weighted Images with 40% inhomogeneity and 9% noise slices 93 and 120.

Methods	Slice 90		Slice 120	
	DSI (GM)	DSI (WM)	DSI (GM)	DSI (WM)
Standard GMM	78.7	90.7	83.2	87.6
Proposed Method	83.8	93.9	84.7	89.6

6. Conclusions

A new method for edge preserving segmentation based on Gaussian Mixture Model is presented. For edge preserving smoothing, bilateral filter is used. To incorporate the spatial correlation among the neighboring pixels, the posterior probability is weighted with a Gaussian kernel. Edge Detection for noisy images is also performed as an outcome. The proposed method has been tested on various synthetic and simulated brain images demonstrating the effectiveness of the method both quantitatively and qualitatively. Apart from these, the method is very simple and has less computational cost compared to the similar methods based on GMM in literature.

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