Fractal Structure of the Urban Objects

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Abstract: *With regards to the big agglomerations and especially the cities, the constructed surface has often got an irregular shape, in spite of the efforts of the urbanists to promote more compact shapes. The internal spatial order of the big agglomerations is a matter of the fractal geometry. This article shows the interest of applying the fractal geometry in cities and in networks of communication of the urban zones. Thus, we are going to focus our attention especially on the coarse analysis of the constructions' shape structure which concern the occupation of the soil surfaces (the fractal of surface such as the carpets of Sierpinski), and to scale permitting the analysis of the agglomerations and cities such like the city of Constantine, Algeria.*

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1. Introduction

During the last ten years, the urban phenomenon has produced different plans of cities especially with the shape of the mural enclosures. All the old towns of Algeria are generally girdled by walls in the shape of ramparts. According to the topography of the places, some are built on hills (like the town of Ghardaïa in the south of the country), others are on flat spaces (like the town of El-Oued in the south east) or on flanks of mountains such as those in Kabylie.

In many civilizations, the rectangular shape appeared very early. It is associated with a certain idea of order dictated by religious reasons or political beliefs. It often appears in residential cities, towns of foundations or dormitories cities (new city of El-Khroub since 1980). The regular circular shape is related to a certain idea of a need for safety and defense for both the polygonal and symmetrical shapes. These irregular shapes are very common and depend on the topography of the sites on which cities are set up. The city of Constantine illustrates this case very well. It is perched on a rock carved by the El-Rhumel Wadi in a natural fortress of a height of approximately 780m and constituting a rampart against any intrusion. The shape of enclosure is not primarily linked to the manner of distributing the area of construction, which can exceed the walls or leave virgin surfaces (empty or vacant zones) inside. But it can also extend along the axes of communication (inter alias transport). It exceeds the enclosures near to the entrance doors and in addition virgin spaces can persist inside the enclosure. This shows that the urban growth follows a certain specific dynamic, which demands the need to modify the enclosures in order to integrate the new suburbs or districts outside the enclosure, located on the road and motorway axes.

The process of interaction between the urban natural growth and its planning design shapes the space evolution in spite of the constraints of the urban growth such as the problems of supplies, cleaning, transportation and other problems related to broken topography. The urban development is more and more breaking out of the planning and following irregular and chaotic shapes. The internal hierarchical shapes seem to follow a fractal order or even multi fractal [10, 23].

In Algeria, under the demographic pressure and the anarchic establishment of the private frame, the adjustment of the urban space leads to a chaotic and continuous increasing occupation of the ground and underground of the great agglomerations like Algiers, Oran, Constantine and Annaba. Indeed, in town center constructions of underground civil engineering works such as car parks, buildings with several undergrounds, roadway systems and various networks (transport in exclusive right of way, water collectors, gas pipes, drinking water and electricity) are multiplied. The initiators of these works should take into account its stability and they should also evaluate the incidence of the choices of layout and the techniques of realization on the existing systems. Therefore, a good amount of knowledge of the soils nature, its geometry and its behavior is essential [2, 25]. In this article, we will focus our attention especially on the coarse analysis of the shape's structure of constructions as regards to the occupation of the ground's surfaces (the fractals of surface such as Sierpinski's carpets), when scales permit the analysis of the agglomerations and the cities such as the town of Constantine [9].

2. The Fractal Geometry

In this geometry, certain shapes of different scales of observation have a feature of the fractals [5, 11, 12, 15, 16, 20, 21]. The Euclidean traditional geometry recognizes an isolated point or the whole of the points composing a figure of dimension zero 0D, that an ordinary line constitutes a figure with a dimension 1D, a surface a shape with two dimensions 2D, etc. Nevertheless, for geometrical objects generated by a reiterated application, such as the curve of Von Koch, its construction requires an initiator (see part 1 of Figure 1), a generator (part 2 of the same figure). It is subjected to a reiterated application which is defined by the generator.

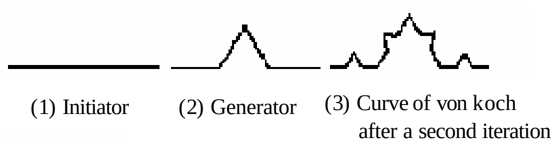

Figure 1. The curve of von koch.

3. Carpets of Sierpinski

The first three iterations of a Sierpinski's carpet on the basis of square with factor of reduction *Error!* and a number of elements (number of black boxes to each iteration with length on side of the generated black boxes L= R*L) N= 5 are illustrated on Figure 2. Knowing that the length of the starting square is L for the application to the cities, it is more interesting to use this type of fractal for the progressive occupation of a surface. Figure 2 illustrates this vision well. Here, the Initiator is a rectangle of side L reduced by the factor *Error!* and N= 5 squares which are placed in such way that a square is in the medium of the initial square, a square with the low right corner and the three others touch it with the left top corner. The internal dimension of homothetic rises by definition with: [8, 19].

Error! represents the fractal dimension. We note that the value lies between 1 and 2. That is to say that the fractal does not cover the entire surface and that there are virgin or white zones called gaps. Of this formula, we retain the following elements:

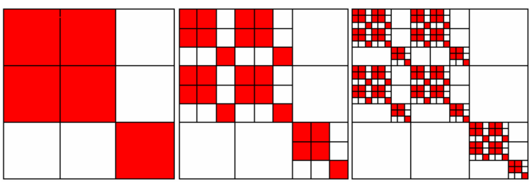

Figure 2. Example of a Sierpinski's carpet.

- R= Factor of reduction.
- N= Number of elements.
- D_s= Internal dimension of homothetic (dimension fractal).
- S= Index of auto similarity.

$$Log\ (N) + D_s * log\ (R) = 0\ Þ\ Error!,\ with\ 0 < R < 1$$

The generator gives additional information influencing the fractal distribution of gaps. This type of fractal is different from a checkerboard. This latter is made of black boxes which are distributed on the surface in a homogeneous way. There are as many white boxes as black ones. All the boxes have the same dimension. As regards to the fractal, the generator thus gives additional information influencing the distribution of the gaps so the description of the fractal aspect as shown in Figure 3.

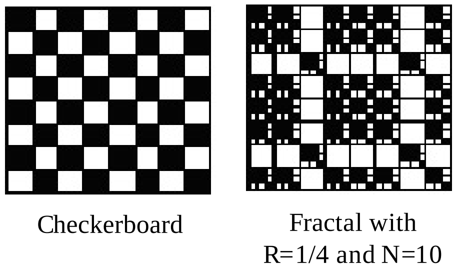

Figure 3. Example of a checkerboard of play of failure and of a Sierpinski's carpet.

Knowing that the fractal consists of a certain level of iteration of black squares of a certain dimension:

$$L_N = R^N * L$$

These squares are laid out in a no homogeneous way giving place to a hierarchy of empty squares (or white), which results from the reiterated application which leads to a progressive occupation of surface. The fractals, permitting the generation of Sierpinski's carpets, have only squares or rectangles as shapes of construction. We can have triangles, circles or polygons. As on Figures 2 and 3, these kinds of Sierpinski's carpet make it possible to represent the shape of the urban objects such as the small islands around the crossroads or big places like parks. We notice that in the case of the carpets of Sierpinski, when R tends towards zero (0) and necessarily N (Number of elements) tends towards the infinite one, virgin spaces called gaps increase and tend to cover all the surface of study [1, 23].

4. Structures Fractals of the Cities

The master line of installation in town planning (plan of development and urban planning) gives the local authorities an instrument of space planning of installation and urban management for harmonious development of the municipality under study and the definition for the considered territory of the general density, the dominant activity, the localization of the equipment of general interest and the constraints which burden this territory. The Soils Occupation Plan (SOP) highlights the general provisions and the constraints for the land use in urbanized or virgin sites. The study on a virgin SOP site makes it possible to determine structural, morphological and architectural solutions for proposals or creations of new urban fabrics which can be related to zones of integrated habitat, zones of activities, industrial parks and zones of tourism expansion. On the other hand, the study on a SOP urbanized site is undertaken only for operations on existing urban fabric for better controlling the urbanization and regulating the rights of land use established in accordance with the provisions of the plan of development and urban planning, to which it refers. Several types of SOP are presented in this case:

- SOP restoration: to demolish then to rebuild according to same typology.
- SOP rehabilitation: to improve the conditions of organization, use and operation.
- SOP reorganization: to remove and change of activities.
- SOP restoration: to safeguard the historical inheritance.
- SOP revalorization: to improve virgin spaces in an urbanized territory and to create new tissues [11, 14, 17, 18].

Certain properties of the fractal geometry resemble the structures of the great agglomerations. It should be known that an urban object (equipment, habitat, etc.) occupies a portion of surface on ground or on water. The urban disposition becomes significant since it narrowly adheres to the network of circulation or the transportation ways. The analysis of the occupation of the grounds and the distribution of the area of construction raise problems related to the levels of observation [6, 13]. In other words, the cartographic scale of representation corresponds to various qualities of information.

Thus, a cartographic representation on a scale of the locality or municipality (regional) will contain certain information concerning the space distribution of the great agglomerations and on the shape of their extension. On the level of districts or units of vicinity, we have information on the site of the urban objects such as constructions, main roads of circulation, parks and great places or crossroads. If one goes down to a more detailed level from the district, then there will be a more precise sight of the small islands with lanes, small public places, walkways, etc. The above paragraphs posses the concept of the fractals, which consists in having a comparable shape to various levels of observation. It corresponds to the most significant property of the fractals, according to which all scales have the same elements of a structure with a reduced dimension (it is the concept of internal homothetic).

The space distribution of constructions in the cities reveals like dense and tight concentrations of urban objects (for example, social buildings, equipment, etc.) leaving only a minimum of virgin or open spaces for the circulation of the people and the goods. With the scale of the locality or municipality, a city will be represented by a point. While going down on a more detailed scale, the large corridors of transportation routes (transport) like the road, motorway and railways appear as well as the waterways.

On the level of the districts, we discover even more virgin spaces in the shape of parks, broad streets and big places. In the interior of small blocks, there are virgin spaces even smaller such as walkways, small public places, lanes, gardens etc. As mentioned before the sierpinsky carpets lend themselves well to the description of the distribution areas of constructions (urban objects). What is known in geometry as the occupation of surfaces is in geography the occupation of the grounds. To emphasize this similarity, we will take an example of a fractal city which follows such a method of construction (see Figure 4). To each iteration corresponds a more or less detailed level of the fractal city as initiator, we took a rectangle. The generator consists of N= 8 rectangles, whose sides length was reduced by a factor (*Error!*). The first iteration of city construction can wrongly be interpreted as a representation of the surface of a city. The second iteration will detail even more the dark rectangle of Figure 4, in a different arrangement from the urban objects created with a different distribution. We can observe the structure of a building block, with big places, wide streets, parks, etc. The third iteration gives a more precise sight of a block with smaller blocks containing lanes, walkways, small places, gardens, etc. which remind us of cadastral maps of urban blocks level.

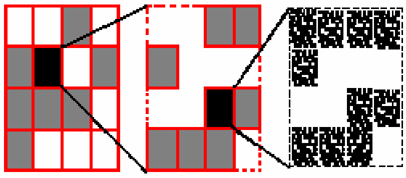

Figure 4. Example of fractal city construction.

5. The Fractal Shape of the Great Agglomerations

The ground is a surface destined to receive an urban assignment and at best a support of infrastructure schematized by the tool of the urban intervention which is the plan. The SOP makes it possible to define the assignment of each piece of ground and the organization of urban fabric in spaces reserved for public equipment, forest or green spaces, agricultural, habitats, as well as urban zones already equipped or about to be equipped (habitat, offices, activities, commerce, etc). Regarding the analysis of the occupation of the grounds and organization of the land use, according to [2], the needs for a city are articulated around the management of space and networks: the cadastral data in general are on a scale of $1/1000^e$ and the topological data on a finer scale $1/200^e$ and $1/500^e$. Do big agglomerations follow a different geometry? We note that the occupation of the ground in the great agglomerations does not follow laws of the traditional geometry known as Euclidean, where the shapes are well defined. The jagged morphology of a great number of urban fabrics seems to follow an organization of the fractal type as Figure 5 shows. Moreover, if we take the Constantine plan (scale $1/7000^e$) Figure 5, we will see blots, which correspond to city constructed areas. It is noted that the surface of the frame resembles a fractal [3, 21]. In Figures 5 and 6, we show a fractal shape dilution of virgin spaces on surface. Moreover, we can generate a fractal which will resemble this shape. The black blots represent the organization of the occupation of space either by constructions, or by the parks, etc.

Figure 5. The map of old Constantine's city.

The radial approach is a local approach. It permits to measure the fractal behavior in the neighborhood of one chosen point, for example the barycentre of an aggregation. The old city of Constantine perched on its rock as shown on the Figure 6 concentrates a strong density of population and the administrative seats of the city (county headquarters, Town Hall, Medersa, high schools, Mosques, etc.). Seen its geographical situation (limited by the stuff the Rhumel River), the urban extension of Constantine is being made to the southeast toward the cities of El-Khroub and to the southwest toward Ain Smara. The picture, on which the redial analysis is made, is an aerial picture dating from 1968. We will specify that the results of the radial analysis depend extensively on the quality of the classification of the satellite or aerial pictures used. Thus, the use of a picture badly classified (for example: the roads and houses belong to the same class whereas in principle these are two different classes) can drive to incoherencies in the results. Several analyses achieved from the city center as shown in Figure 7, give more or less the same results. The radial dimension varies in a fork of 1.80 to 1.85 enhancing the homogeneity of the structure.

Figure 6. The old town of Constantine to the $1/7000^{2nd}$ and on the chart as well as the image, we notice the compartmental aspect of the occupation of spaces of the surface of the ground of the municipality of Constantine (The old rock of Constantine).

This geometry characteristic is that which is appropriate for the description of the area of construction on the level of the great agglomerations and the analysis of the process of the urban growth. The fractal geometry shows the existence of an internal hierarchy in these structures, that describes effects of hierarchization (agglomerations, communes or localities, and small islands), which are typical internal organization of the cities. Three types of fractal dimension are determined [9]:

- The radial dimension (D_r).
- The dimension of the squaring (D_q).
- And the dimension of correlation (D_c).

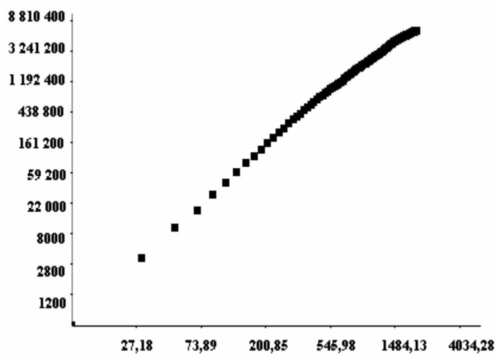

Figure 7. Aerial picture classified of the cloth of the old Constantine city showing chosen four points of numbering like examples for the radial analysis from the downtown. (Curve (D_r= 1, 85) and occupied surface).

5.1. Dimension and Radial Analysis

When the shape of an urban object is compartmental, we carry out a radial dilution. From the center, we determine radial dimension D_r that consists in carrying out a counting of the number of elements inside a circle with variable ray. The barycentre of the shape is selected like starting point.

Error!:

$$N(r)*r^{-D_r} = 1 \text{ what gives } \log(N(r)) = D_r*\log(r)$$

The radial analysis is a method of analyzing the dilution of the area of the construction in the agglomerations and the metropolitan zones [9]. We choose a point of numbering that must be an occupied point; then we surround this point by a square of which the side bi is enlarged gradually so that we get to the step i:

$$b_i = 2*i + 1$$

We determine to every stage, the total number of points occupied N (bi) inside the square. The law fractal takes the shape then of:

$$N(b_i) = b_i^{D_r} = (2*i+1)^{D_r}$$
$$\operatorname{Log} N(b_i) = D_r*\log(b_i)$$

From this linear relation, we determine the radial dimension D_r.

5.2. Dimension and Squaring Analysis

It is a question of covering the shape studied by a squaring whose spacing of the meshes (ε) is variable. Indeed, for each value of (ε) chosen, we carry out the calculation of the number of squares containing the elements of the studied shape. The following relation is then obtained:

$$N(\varepsilon)*\varepsilon^{D_q} = Const$$
$$\operatorname{Log}(N(\varepsilon)) = \operatorname{Log}(Const) - D_q*\operatorname{Log}(\varepsilon)$$

The analysis of the squaring measures the distribution of the construction's area in the whole of the selected window. The area constructed shapes dense sets made up of dense subsets, etc. showing an internal homothety hence the fractality of the urban objects having a no homogeneous distribution. It appears by a hyperbolic distribution of virgin spaces as in the carpets of Sierpinski. More the value of dimension tends towards amount two, more the distribution becomes homogeneous.

5.3. Dimension and Correlation Analysis

It is difficult to determine. The integral of I (correlation ε) is as follows:

$$I(\varepsilon) = Const\,\varepsilon^{-D_c}$$
$$\operatorname{Log} I(\varepsilon) = Const - D_c*\log(\varepsilon)$$

Error!

The function of C (correlation ε) is related to its Integral. *Error!*

$$\operatorname{Log}(C(e)) = \log(Const) - \operatorname{Log}(D_c) - (D_c+1)*\log(e)$$

5.4. Factors Influencing the Results

The factors which can influence the results are:

- The dimension and the position of the window chosen during the analysis of the squaring and correlation [9].
- The resolution used at the time of digitalization [4, 5].
- And the choice of the cartographic representation [12, 13].

5.4.1. Influence of the Window

The first decision to be taken during the analysis is to fix the position and dimensions of the window, in which the determination of fractals dimensions is carried out. Indeed, radial dimension emphasizes an initial place of counting. The choice of the point is very significant. Furthermore, the choice of the barycentre of the urban mass as initial place makes it possible to stabilize the analysis. The dimension of the squaring measures a total property in the window. It can vary according to the size of the chosen window, if it is about a multi fractal. The dimension of correlation is the average value of radial dimension in the surface defined by the window.

5.4.2. Influence of Digitalization

The analysis of the squaring begins with a coarse sight and is directed towards a very fine analysis by using an increasingly tight grid. The natural limit is the pixel. According to whether there is a coarse representation or not, we can ignore or lose more or less large details, which can influence the analysis and the interpretation of the results [5, 11].

5.4.3. Influence of Cartographic Base

A coarser representation of the studied structure affects the value of the fractal dimension. We find here, the influence of the scales of representation which answer specific applications according to qualities of desired information in a given field of study. The remote detection does not have the same concerns that the analysis of the occupation of the pieces on the level of the land register or cadastre [7].

5.4.4. Influence of Topography

We can put the question at which point topography can influence the superficial structure built and thus its fractality. A city of the littoral tends to extend along the coast. It is not possible to build houses on the slopes of cliffs or a mountain. In the oases, constructions of the houses follow the network of the water points [11, 12]. Constantine is a city perched on rock whose limits are encircled by the Rhumel River as shown on Figure 5. That limits the urban development of the city.

6. Conclusion

This article has specially tried to draw the attention to the coarse analysis of the structure of the shape of constructions as regards occupation of surfaces of the ground (fractals of surface such carpets of Sierpinski), and this on scales allowing the analysis of the great agglomerations and the cities such as the town of Constantine. With a rigorous analysis, we can better simulate the organization of the future occupation of spaces to be urbanized. The fractal geometry will make it possible to help the town planners to delimit these spaces and to contribute to the control of the comprehension of their extensions.

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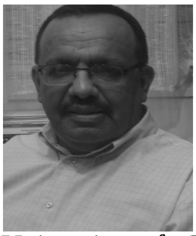

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