A General Characterization of Representing and Determining Fuzzy Spatial Relations

Luyi Bai[1] and Li Yan[2]
[1]College of Information Science and Engineering, Northeastern University, China
[2] College of Computer Science and Technology, Nanjing University of Aeronautics and Astronautics, China

Abstract: *A considerable amount of fuzzy spatial data emerged in various applications leads to investigation of fuzzy spatial data and their fuzzy relations. Because of complex requirements, it is challenging to propose a general fuzzy spatial relationship representation and a general algorithm for determining all fuzzy spatial relations. This paper, presents a general characterization of representing fuzzy spatial relations assuming that fuzzy spatial regions are all fuzzy. On the basis of it, correspondences between fuzzy spatial relations and spatial relations are investigated. Finally, a general algorithm for determining all fuzzy spatial relations is proposed.*

Keywords: *Fuzzy spatial data, fuzzy point, fuzzy line, fuzzy region, fuzzy spatial relations.*

Received May 15, 2012; accepted March 17, 2014; published online March 8, 2015

1. Introduction

Spatial relations play a fundamental role in various application areas, ranging from Geographic Information System (GIS) systems [10] to image understanding [6]. It can be divided into topological (e.g., “overlap”, “meet”, etc.,), directional (e.g., “north of”, “south of”, etc.,) and metric (e.g., “3km away from”, etc.,) relations [2]. Due to topological relations having great significances to spatial reasoning and spatial analysis, it has received increasing attentions in a quantitative way: Both the Region Connection Calculus (RCC) [8] and the 9-intersection model [3] provide a formal characterization of qualitative spatial relations.

However, spatial data is usually fuzzy in the real world applications since their values are subjective in real applications [1, 7]. Thus, the problems that emerge are how fuzzy spatial data should be modeled to determine their fuzzy topological relations [13]. To fill this gap, various definitions of spatial relations between fuzzy spatial data extend either the RCC or the 9-intersection model by considering a *FR* as being composed of two components [9]: One consists of the points that are definitely in *FR* and one consists of the points that are definitely not in the fuzzy region.

A straightforward Fuzzification of the definitions in RCC-8 relations is proposed in [5]. Esterline *et al.* [5] presented a fuzzy version of crisp spatial logic developed by Randell *et al.* [8] that takes the single relation connected-with as primitive. Unfortunately, many properties of the original RCC-8 relations are lost. Moreover, it is unclear how to apply definitions proposed in [5] to calculate the values of the fuzzy spatial relations between two given fuzzy regions. In order to solve these problems, Schockaert *et al.* [12] extend the RCC by providing generalized definitions of the spatial relations as fuzzy relations, which allows expressing the degree to which a particular spatial relation between two regions holds.

Concerning on fuzzification of 9-intersection model, several researches extend 9-intersection approach based on the interior, boundary and exterior of the simple fuzzy spatial regions [14, 15, 16]. Tang *et al.* [16] studied definitions in fuzzy boundary and their relations and then extend the 9-intersection approach to the 3*3 intersection matrix. Furthermore, 4*4 intersection matrixes are formalized based on different topological parts of two fuzzy sets in [14]. In [15] a framework for dealing with fuzzy spatial objects was theoretically proposed, which was also compatible with non-fuzzy spatial object.

However, to our best knowledge, there are less reports on correspondences between fuzzy spatial relations and spatial relations more specifically from mathematical point of view although fuzzy spatial relations have been formalized in the fields of both RCC [5, 12] and 9-intersection model [14, 15, 16] and less reports on general algorithm for determining all fuzzy relations (there are totally 23 fuzzy relations and it will be presented in the later section in this paper) although specific methods of that [12, 16] has been proposed. In this paper, we propose a general characterization of representing and determining fuzzy spatial relations from mathematical point of view assuming that fuzzy spatial regions are all fuzzy. We firstly present the basics of representation of fuzzy spatial topological relations from three aspects: Fuzzy Point (FP), Fuzzy Line (FL) and Fuzzy Region (FR) and then give definitions of fuzzy relations. On this basis, correspondences between fuzzy spatial topological relations and spatial topological relations are investigated. Finally, a general algorithm for determining fuzzy relations is proposed.

The rest of the paper is organized as follows: Section 2 presents basics of representation of fuzzy spatial data and gives definitions of fuzzy spatial topological relations. Section 3 investigates correspondences between fuzzy spatial relations and spatial relations. Section 4 proposes a general algorithm for determining fuzzy relations and Section 5 concludes the paper.

2. Representation of Fuzzy Spatial Topological Relations

In this section, we present the basics of representation of fuzzy spatial topological relations from three aspects, which are *FP*, *FL* and fuzzy region. Fuzzy relations, which will be mentioned below, indicate fuzzy spatial topological relations for simplicity.

2.1. Fuzzy Point

A *FP* is a point whose exact position is not determined but possible positions are known within a certain area. In that case, a *FP* can be viewed as a point in two-dimensional Euclidean space using a membership degree, which returns the value of its membership function, to indicate possible positions of *FP*.

- ***Definition* 1.** (*FP*): For a *FP* (denoted as *FP*), we have $FP = (x, y, \delta)$, including:
  - x is the projection value of the position to *x-axis*.
  - y is the projection value of the position to *y-axis*.
  - δ is the membership degree of the point being the position (x, y) (denoted as $\delta_{(x, y)}$), where $0 \le \delta \le 1$.

A *FP* (x_0, y_0, δ_0) indicates that the possibility of the point locating at (x_0, y_0) is δ_0. For example, a *FP* (2, 5, 0.8) indicates the possibility of the point locating at (2, 5) is 0.8. It is noted that if $\delta_{(x, y)}$=1 and 0 otherwise, the point (x, y, δ) is a crisp one.

The fuzzy relations of two *FP*s contain two cases, which are fuzzy equal (denoted as *FPPequal*) and fuzzy disjoint (denoted as *FPPdisjoint*). In the following definition, we will define each of them. Here, we introduce a mathematical symbol *supp*, where $supp\ A=\{u \mid u \in U, A(u) > 0\}$.

- ***Definition* 2.** Fuzzy Relations of *FP*s: For two *FP*s $FP_1=(x_1, y_1, \delta_1)$ and $FP_2=(x_2, y_2, \delta_2)$, we have:
  - *FPP equal* (FP_1, FP_2): $\exists(supp\ (x_1, \delta_1)=supp(x_2, \delta_2) \wedge supp(y_1, \delta_1)=supp(y_2, \delta_2))$.
  - *FPPdisjoint* (FP_1, FP_2): $\forall(\neg\ (supp(x_1, \delta_1)=supp\ (x_2, \delta_2) \wedge supp(y_1, \delta_1)=supp(y_2, \delta_2)))$.

2.2. Fuzzy Line

A *FL* is a line whose exact position or length is unknown but the area the line ranges is known. The semantic of a line is a point set between two ending points. Accordingly, a *FL* can be viewed as a line in two-dimensional Euclidean space using two membership degrees, which return values of two ending points' membership functions, to indicate possible positions of the fuzzy line.

- ***Definition* 3.** Fuzzy Line: For a *FL* (denoted as *FL*), we have $FL=(x_l, y_l, \delta, x_r, y_r, \delta')$, including:
  - x_l and y_l are the minimum projection values of the *FL* to *x-axis* and *y-axis* (left ending point).
  - x_r and y_r are the maximum projection values of *FL* to *x-axis* and *y-axis* (right ending point).
  - δ and δ' are the membership degrees of the two ending points, where $0 \le \delta \le 1$ and $0 \le \delta' \le 1$.

Since, *FL* is determined by two fuzzy ending points, the membership degree of *FL* is actually determined by the membership degrees of those two ending points.

The fuzzy relations of a *FP* and a *FL* contain three cases, which are fuzzy meet (denoted as *FLPmeet*), fuzzy contain (denoted as *FLPcontain*), and fuzzy disjoint (denoted as *FLPdisjoint*). Definitions of them are given in the following.

- ***Definition* 4.** Fuzzy Relations of *FP* and Fuzzy Line: For a *FP*, $FP=(x_0, y_0, \delta_0)$ and a *FL*, $FL=(x_l, y_l, \delta, x_r, y_r, \delta')$, we have:
  - *FLPmeet* (*FL*, *FP*): $\neg supp\ (x_0, y_0, \delta_0) \cap supp\ (x_l, y_l, \delta) = \varnothing \vee \neg supp\ (x_0, y_0, \delta_0) \cap supp\ (x_r, y_r, \delta') = \varnothing$.
  - *FLPcontain* (*FL*, *FP*): $\begin{vmatrix} x_l & y_l & 1 \\ x_r & y_r & 1 \\ x_0 & y_0 & 1 \end{vmatrix} = 0 \wedge (supp\ (\min(x_l, x_r), \delta) < supp\ (x_0, \delta_0) < supp\ (\max(x_l, x_r), \delta) \vee (supp\ (\min(y_l, y_r), \delta) < supp\ (x_0, \delta_0) < supp\ (\max(y_l, y_r), \delta))$.
  - *FLPdisjoint* (*FL*, *FP*): $\neg$ (*FLPmeet* (*FL*, *FP*)$\vee$ *FLPcontain*(*FL*, *FP*))$\Leftrightarrow$*FP* stays in the left of *FL*$\Leftrightarrow$ $\begin{vmatrix} x_l & y_l & 1 \\ x_r & y_r & 1 \\ x_0 & y_0 & 1 \end{vmatrix} > 0$ denoted *FLPleft* $(FP, \delta_0, FL, \delta, \delta')$$\vee$*FP* stays in the right of *FL*$\Leftrightarrow$ $\begin{vmatrix} x_l & y_l & 1 \\ x_r & y_r & 1 \\ x_0 & y_0 & 1 \end{vmatrix} < 0$ denoted as *FLPright* $(FP, \delta_0, FL, \delta, \delta')$.

The fuzzy relations of two fuzzy lines contain six cases, which are fuzzy intersect (denoted as *FLLintersect*), fuzzy equal (denoted as *FLLequal*), fuzzy contain (denoted as *FLLcontain*), fuzzy overlap (denoted as *FLLoverlap*), fuzzy meet (denoted as *FLLmeet*), and fuzzy disjoint (denoted as *FLLdisjoint*).

- ***Definition* 5.** Fuzzy Relations of Fuzzy Lines: For two fuzzy lines $FL_1=(x_{1l}, y_{1l}, \delta_1, x_{1r}, y_{1r}, \delta_1')$ and $FL_2=(x_{2l}, y_{2l}, \delta_2, x_{2r}, y_{2r}, \delta_2')$, we have:
  - *FLLintersect* (FL_1, FL_2): $((FLPleft\ (x_{1l}\ y_{1l}, \delta_1, FL_2) \wedge FLPright\ (x_{1r}, y_{1r}, \delta_1', FL_2)) \vee FLPleft\ (x_{1r}, y_{1r}, \delta_1', FL_2) \wedge FLPright\ (x_{1l}, y_{1l}, \delta_1, FL_2)) \wedge ((FLPleft\ (x_{2l}, y_{2l}, \delta_2, FL_1) \wedge FLPright\ (x_{2r}, y_{2r}, \delta_2', FL_1)) \vee (FLPleft\ (x_{2r}, y_{2r}, \delta_2', FL_1) \wedge FLPright\ (x_{2l}, y_{2l}, \delta_2, FL_1)))$.

- *FLLequal* (FL_1, FL_2): *FLPmeet* (FL_2, x_{1l}, y_{1l}, δ_1) $\wedge$ *FLPmeet* (FL_2, x_{1r}, y_{1r}, δ_1').
- *FLLcontain* (FL_1, FL_2): (*FLPcontain* (FL_1, x_{2l}, y_{2l}, δ_2) $\wedge$ ($\neg$*FLPdisjoint* (FL_1, x_{2r}, y_{2r}, δ_2'))) $\vee$ (*FLPcontain* (FL_1, x_{2r}, y_{2r}, δ_2') $\wedge$ ($\neg$*FLPdisjoint* (FL_1, x_{2l}, y_{2l}, δ_2))).
- *FLLoverlap* (FL_1, FL_2): (*FLPcontain* (FL_1, x_{2l}, y_{2l}, δ_2) $\vee$ *FLPcontain* (FL_1, x_{2r}, y_{2r}, δ_2')) $\wedge$ (*FLPcontain* (FL_2, x_{1l}, y_{1l}, δ_1) $\vee$ *FLPcontain* (FL_2, x_{1r}, y_{1r}, δ_1')).
- *FLLmeet* (FL_1, FL_2): ((*FLPcontain* (FL_1, x_{2l}, y_{2l}, δ_2) $\vee$ *FLPcontain* (FL_1, x_{2r}, y_{2r}, δ_2') $\vee$ *FLPcontain* (FL_2, x_{1l}, y_{1l}, δ_1) $\vee$ *FLPcontain* (FL_2, x_{1r}, y_{1r}, δ_1')) $\wedge$ ($y_{1r}-y_{1l}$)/ ($x_{1r}-x_{1l}$) $\neq$ ($y_{2r}-y_{2l}$)/ ($x_{2r}-x_{2l}$)) $\vee$ ((*FLPmeet* (FL_1, x_{2l}, y_{2l}, δ_2) $\vee$ *FLPmeet* (FL_1, x_{2r}, y_{2r}, δ_2') $\vee$ *FLPmeet* (FL_2, x_{1l}, y_{1l}, δ_1) $\vee$ *FLPmeet* (FL_2, x_{1r}, y_{1r}, δ_1')) $\wedge$ $\neg$ (*FLLequal* (FL_1, FL_2) $\vee$ *FLLcontain* (FL_1, FL_2) $\vee$ *FLLcontain* (FL_2, FL_1))).
- *FLLdisjoint* (FL_1, FL_2): *FLPdisjoint* (FL_2, x_{1l}, y_{1l}, δ_1) $\wedge$ *FLPdisjoint* (FL_2, x_{1r}, y_{1r}, δ_1') $\wedge$ *FLPdisjoint* (FL_1, x_{2l}, y_{2l}, δ_2) $\wedge$ *FLPdisjoint* (FL_1, x_{2r}, y_{2r}, δ_2') $\wedge$ $\neg$ (*FLLintersect* (FL_1, FL_2) $\vee$ *FLLcontain* (FL_1, FL_2) $\vee$ *FLLcontain* (FL_2, FL_1)).

2.3. Fuzzy Region

A general definition descries a crisp region as a set of disjoint, connected components, called faces, possibly with disjoint holes [4, 11] in the Euclidean space IR^2. By analogy with the generalization of crisp regions to fuzzy regions, we strive for fuzzy regions on the basis of the point set paradigm and fuzzy concepts. For simplicity, we only talk about two-dimensional regions without holes.

A *FR* is a region with indeterminate boundaries. Fuzzy regions can be represented by *MBR* (minimum bounding rectangle) so that we can use two *FP*s to represent fuzzy regions.

- ***Definition* 6.** Fuzzy Region: For a *FR* (denoted as *FR*), we have $FR=(x_{min}, y_{min}, \delta, x_{max}, y_{max}, \delta')$, including:
  - x_{min} and y_{min} are the minimum projection values of the *FR* to *x-axis* and *y-axis* (lower left ending point).
  - x_{max} and y_{max} are the maximum projection values of *FR* to *x-axis* and *y-axis* (upper right ending point).
  - δ and δ' are the membership degrees of the above two representing points, where $0<\delta\leq 1$ and $0<\delta'\leq 1$.

Similar as the fuzzy line, the membership degree of *FR* is determined by the membership degrees of the two representing ending points.

The fuzzy relations of a *FP* and a *FR* contain three cases, which are fuzzy disjoint (denoted as *FRPdisjoint*), fuzzy meet (denoted as *FRPmeet*) and fuzzy contain (denoted as *FRPcontain*). Definitions of them are given in the following.

- ***Definition* 7.** Fuzzy Relations of *FP* and Fuzzy Region: For a *FP*, $FP=(x_0, y_0, \delta_0)$ and a *FR*, $FR=(x_{min}, y_{min}, \delta, x_{max}, y_{max}, \delta')$, we denote four *FL* of the fuzzy region: $FL_1=(x_{min}, y_{max}, \delta_1, x_{min}, y_{min}, \delta_1')$, $FL_2=(x_{min}, y_{min}, \delta_2, x_{max}, y_{min}, \delta_2')$, $FL_3=(x_{max}, y_{min}, \delta_3, x_{max}, y_{max}, \delta_3')$, $FL_4=(x_{max}, y_{max}, \delta_4, x_{min}, y_{max}, \delta_4')$. Then, we have:
  - *FRPdisjoint* (*FP*, *FR*): $\neg$(*FLPleft* (FL_1, *FP*) $\wedge$ *FLPleft* (FL_2, *FP*,) $\wedge$ *FLPleft* (FL_3, *FP*) $\wedge$ *FLPleft* (FL_4, *FP*)).
  - *FRPmeet* (*FP*, *FR*): *FLPmeet* (FL_1, *FP*) $\vee$ *FLPmeet* (FL_2, *FP*) $\vee$ *FLPmeet* (FL_3, *FP*)$\vee$ *FLPmeet* (FL_4, *FP*)$\vee$ *FLPcontain* (FL_1, *FP*)$\vee$ *FLPcontain* (FL_2, *FP*)$\vee$ *FLPcontain* (FL_3, *FP*) $\vee$ *FLPcontain* (FL_4, *FP*).
  - *FRPcontain* (*FP*, *FR*): *FLPleft* (FL_1, *FP*) $\wedge$ *FLPleft* (FL_2, *FP*) $\wedge$ *FLPleft* (FL_3, *FP*) $\wedge$ *FLPleft* (FL_4, *FP*).

The fuzzy relations of a *FL* and a *FR* contain four cases, which are fuzzy contain (denoted as *FRLcontain*), fuzzy intersect (denoted as *FRLintersect*), fuzzy meet (denoted as *FRLmeet*) and fuzzy disjoint (denoted as *FRLdisjoint*). Definitions of them are given in the following.

***Definition* 8.** Fuzzy Relations of *FL* and Fuzzy Region: For a *FL*, $FL=(x_l, y_l, \delta, x_r, y_r, \delta')$ and a *FR*, $FR=(x_{min}, y_{min}, \delta, x_{max}, y_{max}, \delta')$, we denote four *FL* of the fuzzy region: $FL_1=(x_{min}, y_{max}, \delta_1, x_{min}, y_{min}, \delta_1')$, $FL_2=(x_{min}, y_{min}, \delta_2, x_{max}, y_{min}, \delta_2')$, $FL_3=(x_{max}, y_{min}, \delta_3, x_{max}, y_{max}, \delta_3')$, $FL_4=(x_{max}, y_{max}, \delta_4, x_{min}, y_{max}, \delta_4')$. Then, we have:

- *FRLcontain* (*FR*, *FL*): (*FRPcontain* (*FR*, x_l, y_l, δ)$\wedge$ $\neg$ *FRPdisjoint* (*FR*, x_r, y_r, δ'))$\vee$(*FRPcontain* (*FR*, x_r, y_r, δ')$\wedge$ $\neg$ *FRPdisjoint* (*FR*, x_l, y_l, δ)).
- *FRLintersect* (*FR*, *FL*): *FLLintersect* (*FL*, FL_1)$\vee$ *FLLintersect* (*FL*, FL_2)$\vee$*FLLintersect* (*FL*, FL_3)$\vee$ *FLLintersect* (*FL*, FL_4)$\vee$((*FRPcontain* (*FR*, x_l, y_l, δ) $\wedge$*FRPdisjoint* (*FR*, x_r, y_r, δ'))$\vee$ (*FRPcontain* (*FR*, x_r, y_r, δ')$\wedge$*FRPdisjoint*(*FR*, x_l, y_l, δ))).
- *FRLmeet* (*FR*, *FL*): (($\neg$ *FRPcontain* (*FR*, x_l, y_l, δ) $\wedge$ $\neg$ *FRPcontain* (*FR*, x_r, y_r, δ'))$\wedge$(*FLLmeet* (*FL*, FL_1) $\vee$ *FLLmeet* (*FL*, FL_2)$\vee$*FLLmeet* (*FL*, FL_3)$\vee$ *FLLmeet* (*FL*, FL_4)$\vee$*FLLoverlap* (*FL*, FL_1)$\vee$ *FLLoverlap* (*FL*, FL_2)$\vee$*FLLoverlap* (*FL*, FL_3)$\vee$ *FLLoverlap* (*FL*, FL_4)$\vee$*FLLcontain* (*FL*, FL_1) $\vee$ *FLLcontain* (FL_1, *FL*)$\vee$*FLLcontain* (*FL*, FL_2) $\vee$ *FLLcontain* (FL_2, *FL*)$\vee$*FLLcontain* (*FL*, FL_3) $\vee$ *FLLcontain* (FL_3, *FL*)$\vee$*FLLcontain* (*FL*, FL_4) $\vee$ *FLLcontain* (FL_4, *FL*)).
- *FRLdisjoint* (*FR*, *FL*): (*FRPdisjoint* (*FR*, x_l, y_l, δ)$\vee$ *FRPdisjoint* (*FR*, x_r, y_r, δ'))$\wedge$ $\neg$ *FRLintersect* (*FR*, *FL*) $\wedge$ $\neg$ *FRLmeet* (*FR*, *FL*).

The fuzzy relations of two fuzzy regions contain five cases, which are fuzzy equal (denoted as *FRRequal*), fuzzy contain (denoted as *FRRcontain*), fuzzy overlap (denoted as *FRRoverlap*), fuzzy meet (denoted as *FRRmeet*) and fuzzy disjoint (denoted as *FRRdisjoint*). Definitions of them are given in the following:

- ***Definition* 9.** Fuzzy Relations of Fuzzy Regions: For two fuzzy regions $FR_1=(x_{1min}, y_{1min}, \delta_1, x_{1max}, y_{1max}, \delta_1')$ and $FR_2=(x_{2min}, y_{2min}, \delta_2, x_{2max}, y_{2max}, \delta_2')$, we denote four *FL* of the fuzzy region: $FL_{11}=(x_{1min}, y_{1max}, \delta_{11}, x_{1min}, y_{1min}, \delta_{11}')$, $FL_{12}=(x_{1min}, y_{1min}, \delta_{12}, x_{1max}, y_{1min}, \delta_{12}')$, $FL_{13}=(x_{1max}, y_{1min}, \delta_{13}, x_{1max}, y_{1max}, \delta_{13}')$, $FL_{14}=(x_{1max}, y_{1max}, \delta_{14}, x_{1min}, y_{1max}, \delta_{14}')$; $FL_{21}=(x_{2min}, y_{2max}, \delta_{21}, x_{2min}, y_{2min}, \delta_{21}')$, $FL_{22}=(x_{2min}, y_{2min}, \delta_{22}, x_{2max}, y_{2min}, \delta_{22}')$, $FL_{23}=(x_{2max}, y_{2min}, \delta_{23}, x_{2max}, y_{2max}, \delta_{23}')$, $FL_{24}=(x_{2max}, y_{2max}, \delta_{24}, x_{2min}, y_{2max}, \delta_{24}')$. Then, we have:
  - *FRRequal* (FR_1, FR_2): *FPPequal* (x_{1min}, y_{1min}, δ_{12}, x_{2min}, y_{2min}, δ_{22}) $\wedge$ *FPPequal* (x_{1max}, y_{1min}, δ_{13}, x_{2max}, y_{2min}, δ_{23}) $\wedge$

FPPequal (x_{1max}, y_{1max}, δ_{14}, x_{2max}, y_{2max}, δ_{24})∧*FPPequal* (x_{1min}, y_{1max}, δ_{11}, x_{2min}, y_{2max}, δ_{21}).

- *FRRcontain* (FR_1, FR_2): ¬ *FRRequal* (FR_1, FR_2)∧ (*FRPcontain* (FR_1, x_{min}, y_{min}, δ_{22}) ∨ *FRPmeet* (FR_1, x_{min}, y_{min}, δ_{22})) ∧ (*FRPcontain* (FR_1, x_{2max}, y_{2min}, δ_{23}) ∨ *FRPmeet* (FR_1, x_{2max}, y_{2min}, δ_{23})) ∧ (*FRPcontain* (FR_1, x_{2max}, y_{2max}, δ_{24})∨ *FRPmeet* (FR_1, x_{2max}, y_{2max}, δ_{24})) ∧ (*FRPcontain* (FR_1, x_{2min}, y_{2max}, δ_{21}) ∨ *FRPmeet* (FR_1, x_{2min}, y_{2max}, δ_{21})).
- *FRRoverlap* (FR_1, FR_2): *FRLintersect* (FR_1, FL_{21})∨ *FRLintersect* (FR_1, FL_{22})∨*FRLintersect* (FR_1, FL_{23})∨ *FRLintersect* (FR_1, FL_{24}).
- *FRRmeet* (FR_1, FR_2): (*FRPmeet* (FR_2, x_{1min}, y_{1min}, δ_{12})∨ *FRPmeet* (FR_2, x_{1max}, y_{1min}, δ_{13})∨ *FRPmeet* (FR_2, x_{1max}, y_{1max}, δ_{14}) ∨ *FRPmeet* (FR_2, x_{1min}, y_{1max}, δ_{11}) ∨ *FRPmeet* (FR_1, x_{2min}, y_{2max}, δ_{21})∨*FRPmeet* (FR_1, x_{2min}, y_{2max}, δ_{21})∨ *FRPmeet* (FR_1, x_{2min}, y_{2max}, δ_{21})∨*FRPmeet* (FR_1, x_{2min}, y_{2max}, δ_{21})) ∧ ¬ (*FRRequal* (FR_1, FR_2)∨*FRRoverlap* (FR_1, FR_2)∨ *FRRcontain* (FR_1, FR_2)∨*FRRcontain* (FR_2, FR_1)).
- *FRRdisjoint* (FR_1, FR_2): (*FRLdisjoint* (FR_1, FL_{21})∧ *FRLdisjoint* (FR_1, FL_{22})∧*FRLdisjoint* (FR_1, FL_{23})∧ *FRLdisjoint* (FR_1, FL_{24})∧¬ *FRRcontain* (FR_1, FR_2))∨ (*FRLdisjoint* (FR_2, FL_{11})∧*FRLdisjoint* (FR_2, FL_{12})∧ *FRLdisjoint* (FR_2, FL_{13})∧*FRLdisjoint* (FR_2, FL_{14})∧¬ *FRRcontain* (FR_2, FR_1)).

3. Correspondences between Fuzzy Spatial Topological Relations and Spatial Topological Relations

In this section, we present correspondences between fuzzy spatial topological relations and spatial topological relations on the basis of studies in the above section. The correspondences come from six cases: Point and point, line and point, line and line, region and point, region and line, region and region.

For fuzzy relations of *FPs*, *FPPequal* is the fuzzy relation if there are possible equal points and *FPPdisjoint* is the fuzzy relation if there are no possible equal points.

For fuzzy relations of a *FP* and a fuzzy line, *FLPmeet* is the fuzzy relation if there is a possible point meeting a possible line; *FLPcontain* is the relation if there is a possible line containing possible points and there is no possible point meeting a possible line; *FLPdisjoint* is the relation if all possible points and all possible lines are disjoint.

For fuzzy relations of two fuzzy lines, *FLLintersect* is the fuzzy relation if there is a possible line of one *FL* intersecting a possible line of the other fuzzy line; *FLLequal* is the fuzzy relation if there are two possible equal lines of two fuzzy lines; *FLLcontain* is the fuzzy relation if the minimum and maximum ending points of a possible line of one *FL* are contained by a possible line of the other fuzzy line; *FLLoverlap* is the fuzzy relation if the maximum ending point of a possible line of one *FL* is contained by a possible line of the other *FL* and the minimum ending point of a possible line of the other *FL* is contained by a possible line of this fuzzy line; *FLLmeet* is the fuzzy relation if there is a ending point of a possible line of one *FL* meeting a ending point of a possible line of the other fuzzy line; *FLLdisjoint* is the fuzzy relation if all the two possible lines of two fuzzy lines are disjoint.

For fuzzy relations of a *FP* and a fuzzy region, it is *FRPdisjoint* if all possible points of the *FP* and all possible regions of *FR* are disjoint; it is *FRPmeet* if there is a possible point of the *FP* meeting a possible region of *FR* and not all possible points of *FP* staying outside or inside all possible regions of the fuzzy region; it is *FRPcontain* if all possible points of *FP* are contained by all possible regions of the fuzzy region.

For fuzzy relations of a *FL* and a fuzzy region, it is *FRLcontain* if all possible lines of *FL* are contained by all possible regions of the fuzzy region; it is *FRLdisjoint* if all possible lines of *FL* and all possible regions of *FR* are disjoint; it is *FRLmeet* if there is a possible line of *FL* meeting a possible region of *FR* and their fuzzy relation is not *FRLcontain* or *FRLdisjoint*; it is *FRLintersect* if there is a possible line of *FL* intersecting a possible region of *FR* and their fuzzy relation is not *FRLcontain* or *FRLdisjoint* or *FRLmeet*.

For fuzzy relations of two fuzzy regions, it is *FRRdisjoint* if all possible regions of one *FR* and all possible regions of the other *FR* are disjoint; it is *FRRcontain* if all possible regions of one *FR* are contained by all possible regions of the other fuzzy region; it is *FRRoverlap* if there is a possible region of one *FR* overlaps a possible region of the other *FR* and their fuzzy relation is not *FRRdisjoint* or *FRRcontain*; it is *FRRmeet* if there is a possible region of one *FR* meeting a possible region of the other *FR* and their fuzzy relation is not *FRRdisjoint*, *FRRcontain* or *FRRoverlap*; it is *FRRequal* if a possible region of one *FR* and a possible region of the other *FR* are equal and their fuzzy relation is not *FRRdisjoint*, *FRRcontain*, *FRRoverlap* or *FRRmeet*.

In order to, present correspondences between fuzzy spatial topological relations and spatial topological relations, Figure 1 shows their correspondences. In Figure 1, it is denoted as cross if there are no correspondences; it is denoted as tick if there are correspondences and the fuzzy relation has the corresponding crisp relation; it is blank if there are correspondences but the fuzzy relation has no corresponding crisp relation.

	FPPequal			FPPdisjoint			FLPmeet			FLPcontain			FLPdisjoint			FLLequal		
1 2 3	√	√	×		√	×	×	√	×	×	√	×	×	√	×	√	√	√
4 5 6	×	×	×	×	×	×	×	√	√	×	√	√	×				√	√
equal (1)	FLLdisjoint			FLLmeet			FLLcontain			FLLoverlap			FLLintersect			FRPdisjoint		
disjoint (2)		√			√		√	√	√	√	√	√		√		×	√	×
					√			√	√		√	√	√	√		×		
overlap (3)	FRPmeet			FRPcontain			FRLcontain			FRLintersect			FRLmeet			FRLdisjoint		
intersect (4)	×	√	×	×		×	×		×	×	√	×	×	√	×	×	√	×
	×	√	√	×		√			√	√	√	√		√				
meet (5)	FRRequal			FRRcontain			FRRoverlap			FRRmeet			FRRdisjoint					
contain (6)	√	√	√	√	√	√	√	√	√		√			√				
	×	√	√	×	√	√	×	√	√	×	√		×					

Figure 1. Correspondences between fuzzy and crisp spatial relations.

4. Determination of Fuzzy Spatial Topological Relations

In this section, we present how to determine fuzzy spatial topological relations. We firstly propose a general algorithm for determining fuzzy relations, and then an example is given to explain it.

There are 23 fuzzy relations. Each of them needs an algorithm to determine fuzzy relations. Since, there are some common points in each of them, we propose a general algorithm. Then, if a certain fuzzy relation is required, the general algorithm can be extended.

The Algorithm 1 is a general algorithm for determining fuzzy relations. It contains two loops in order to compare all possible fuzzy relations. The possibility of the relation employs cumulative way to compute. The finally returned value is divided by all membership degrees because each desired membership degree is obtained by two membership degrees, which are composed of two *FPs*. For that reason, membership degrees should be transformed into relative values.

Algorithm 1: Frelation Y, Z.

1. *for (k = 1; k <= X; k++)*
2. *let* $\delta_k = 0$
3. *end for*
4. *for (m = 1; m <= i; i++)*
5. *for (n = 1; n <= m; n++)*
6. *for (r = 1; r <= X; r++)*
7. *if* F_r*relation (Y, Z)*
8. $\delta_r = \delta_r + \delta_X$
9. *end for*
10. *end for*
11. *end for*
12. *if true (*F_T*relation)*
13. *return* $\delta_T / \sum_{i=1}^{X} \delta i$

For the algorithm, some tips need to be explained: *Y* and *Z* indicate two fuzzy spatial objects (it can be further explained by their representing points); *X* indicates number of fuzzy relations between *Y* and *Z*. F_r*relation* (*Y*, *Z*) compares fuzzy relations *X* times according to definition 2, 4, 5, 7, 8, or 9; *T* indicates the number satisfying fuzzy relation ranging from 1 to *X*, and the returned value is divided by all the possible membership degrees.

In succession, we give an example to describe process of determining fuzzy relations using the proposed algorithm. For convenience, we use the example of two fuzzy regions.

Consider two fuzzy regions FR_1={(1, 2, 0.6, 6, 7, 0.7), (2, 4, 0.4, 7, 8, 0.3)} and FR_2={(6, 3, 0.8, 9, 10, 0.6), (6, 1, 0.2, 9, 3, 0.4)}. According to line 10 in algorithm *FRRrelation*, we can get the fuzzy relation of FR_1 and FR_2 is *FRRoverlap*. The *meet* pair are {(1, 2, 0.6, 6, 7, 0.7), (6, 3, 0.8, 9, 10, 0.6)}, {(1, 2, 0.6, 6, 7, 0.7), (6, 1, 0.2, 9, 3, 0.4)}; the *overlap* pair is {(2, 4, 0.4, 7, 8, 0.3), (6, 3, 0.8, 9, 10, 0.6)}; the *disjoint* pair is {(2, 4, 0.4, 7, 8, 0.3), (6, 1, 0.2, 9, 3, 0.4)}. Possibility of each relation is: δ_4=0+0.6×0.7×0.8×0.6+0.6×0.7×0.2×0.4=0.2352; δ_3=0+0.4×0.3×0.8×0.6=0.0576; δ_5=0+0.4×0.3×0.2×0.4=0.0096. Finally, we get the possibility of each relation: $\delta_4/(\delta_1+\delta_2+\delta_3+\delta_4+\delta_5)\approx$ 0.778; $\delta_3/(\delta_1+\delta_2+\delta_3+\delta_4+\delta_5)\approx$ 0.190; $\delta_5/(\delta_1+\delta_2+\delta_3+\delta_4+\delta_5)$=0.032. Consequently, the possibility that the relation of the two fuzzy regions is *meet* approximately amounts to 0.778; is *overlap* approximately amounts to 0.190; is *disjoint* approximately amounts to 0.032.

5. Conclusions

In order to present a general characterization of representing and determining fuzzy spatial relations, definitions of fuzzy spatial objects and their relations are given. Then, correspondences between fuzzy and crisp spatial relations are investigated. Finally, a general algorithm for determining all fuzzy relations is proposed and a followed example explains it. Compared with other methods, our approaches focus on correspondences between fuzzy spatial relations and spatial relations more specifically from mathematical point of view, which has less been studied. What's more, a general formal algorithm for determining fuzzy relations is proposed, while majority of others are methods for specific domains. Consequently, our approaches can be applied to more applications than others.

In the future, we intend to apply the proposed approaches to spatiotemporal applications. A possible solution is to integrate our approaches with MBR strategies. Another future research topic is extending two-dimensional spatial data to three-dimensional one, and discussing their continuous cases.

Acknowledgements

This work was supported by the National Natural Science Foundation of China (61370075 and 61402087), the Fundamental Research Fund for the Central Universities (N130323006), the Scientific Research Fund of Hebei Education Department (QN2014339), and the Doctoral Fund Project of Northeastern University at Qinhuangdao (XNB201428).

References

[1] Azough A., France F., Delteil A., Hacid S., and De Marchi F., "Fuzzy Conceptual Graphs for Handling Uncertainty in Semantic Video Retrieval," *in Proceedings of the 11th IEEE International Symposium on Multimedia*, USA, pp. 324-329, 2009.

[2] Claramunt C. and Theriault M., "Fuzzy Semantics for Direction Relations between Composite Regions," *Information Sciences*, vol. 160, no. 1-4, pp. 73-90, 2004.

[3] Du S., Qin Q., Wang Q., and Li B., "Fuzzy Description of Topological Relations I: A Unified Fuzzy 9-Intersection Model," *Lecture*

Notes in Computer Science, vol. 3612, pp. 1261-1273, 2005.

[4] Erwig M. and Schneider M., "Vague Regions," *Lecture Notes in Computer Science*, vol. 1262, pp. 298-320, 1997.

[5] Esterline A., Dozier G., and Homaifar A., "Fuzzy Spatial Reasoning," *in Proceedings of the International Fuzzy Systems Association Conference*, Taiwan, pp. 162-167, 1997.

[6] Luo C., Yan H., and Cheng Y., "The Summary on Description of Spatial Relation Image," *in Proceedings of Computational Intelligence and Software Engineering*, Jinan, pp. 1-4, 2010

[7] Prasad N. and Ramakrishna S., "An Efficient Traffic Forecasting System Based on Spatial Data and Decision Trees," *the International Arab Journal of Information Technology*, vol. 11, no. 2, pp. 186-194, 2014.

[8] Randell A., Cui Z., and Cohn G., "A Spatial Logic Based on Regions and Connection," *in Proceedings of the 3rd International Conference on Knowledge Representation and Reasoning*, Morgan Kaufmann, pp. 165-176, 1992.

[9] Roy J. and Stell G., "Spatial Relations between Indeterminate Regions," *International Journal of Approximate Reasoning*, vol. 27, no. 3, pp. 205-234, 2001.

[10] Santosh C., Laurent W., and Bart L., "Using Spatial Relations for Graphical Symbol Description," *in Proceedings of the 20th International Conference on Pattern Recognition*, Istanbul, pp. 2041-2044, 2010.

[11] Schneider M., "Spatial Data Types for Database Systems-Finite Resolution Geometry for Geographic Information Systems," *Lecture Notes in Computer Science*, Springer, 1997.

[12] Schockaert S., Cornelis C., Cock D., and Kerre E., "Fuzzy Spatial Relations between Vague Regions," *in Proceedings of the 3rd International IEEE Conference Intelligent Systems*, London, pp. 221-226, 2006.

[13] Sözer A., Yazici A., Oğuztüzün H., and Tas O., "Modeling and Querying Fuzzy Spatiotemporal Databases," *Information Sciences*, vol. 178, no. 19, pp. 3665-3682, 2008.

[14] Tang X. and Kaina W., "Analysis of Topological Relations between Fuzzy Regions in A General Fuzzy Topological Space," *International Journal of Applied Earth Observation and Geoinformation*, vol. 12, no. 2, pp. 151-165, 2010.

[15] Tang X., Fang Y., and Kainz W., "Fuzzy Topological Relations between Fuzzy Spatial Objects," *in Proceedings of the 3rd International Conference on Fuzzy Systems and Knowledge Discovery*, Xi'.an, pp. 324-333, 2006.

[16] Tang X., Fang Y., and Kainz W., "Topological Matrices for Topological Relations between Fuzzy Regions," *in Proceedings of the 4th International Symposium on Multispectral Image Processing and Pattern Recognition*, Wu Han, pp. 105-122, 2005.

Luyi Bai received his PhD degree from Northeastern University, China. Currently, he is a lecturer at Northeastern University at Qinhuangdao, China. His current research interests include uncertain databases and fuzzy spatiotemporal XML data management. He has published papers in several Journals such as integrated computer-aided engineering and applied intelligence.

Li Yan received her PhD degree from Northeastern University, China. Currently, she is an associate professor at Northeastern University, China. Her research interests include database modeling, XML data management, as well as imprecise and uncertain data processing. She has published some papers International Journals such as Experts Systems with Applications, Integrated Computer-Aided Engineering, Information Systems Frontiers, International Journal of Intelligent Systems and Journal of Intelligent and Fuzzy Systems. She has authored two monographs published by Springer, and published several edited books with Springer and IGI Global.