# Graph-Theoretic Model for Analyzing the Properties of Local Network in Social Internet of Things

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**Abstract:** *The Internet of Things (IoT) performs intensive and varied communications by a large number of devices will be overwhelmed shortly. These devices are expected to provide millions of known services and other new services. This may result in several implications for scalability, navigability, and trustworthiness. That is, it may become challenging for a device to reach an appropriate service provided by other devices. Recently a new paradigm known as the Social Internet of Things (SIoT) has gained momentum with many researchers working towards incorporating social networking concepts into IoT. By utilizing SIoT concepts, humans' innate ability to discover, select, and use services in social networks can be extended to devices participating in IoT networks. In this context, the benchmark dataset for SIoT, namely the Santander city dataset is being adopted by many researchers for validating their proposals. In this paper, an attempt has been made to analyze the Santander dataset using a graph theoretic approach based on the Local Network Structure (LNS) i.e., node or vertex characteristics namely centrality measures. The novelty in this work can be ascribed to the application of the graph theoretic approach to large networks or graphs and get hindsight of certain intrinsic properties of large real-time networks. The major centrality measures considered in this work are degree centrality, eigen Katz centrality, vector centrality, closeness, page rank, and betweenness centralities. The integration of social networking concepts into IoT enhances service discovery, trust management, and scalability by enabling autonomous device relationships, improving network navigability, optimizing resource allocation, and strengthening security. It is observed that the outcome of the experiment provides clear insights into the efficacy of different social relationships on the aforesaid metrics using Local or Node Level analysis say, the Ownership Object Relationship (OOR) relationship displays significant node degree (43.68%). Moreover, the Co-Location Object Relationship (CLOR) relationship exhibits the highest betweenness centrality (85.6%), while the effectiveness of closeness centrality is demonstrated in three relationships: OOR (28.00%), Social Object Relationship (SORv1) (24.14%), and SORv2 (24.45%).*

**Keywords:** *Parental object relationship, SIoT, social object relationship, CLOR, C-WOR, ownership object relationship.*

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## 1. Introduction

Nowadays, the Internet is witnessing a proliferation of connected devices and applications that are heterogeneous and autonomous. This development is called the Internet of Things (IoT). It is considered the evolution of the Internet wherein everyday objects simply referred to as things are getting interconnected. Generally, 'things' refer to a myriad of devices that seek and provide various kinds of services. The physical devices get interconnected to the cyber world using IoT. Recently IoT has been applied in diverse applications like water and air pollution detection systems [15], identification of disease in plants [26], and monitoring of flora and fauna of underwater ecosystems [6]. Smart sensors, RFID tags, smartphones, and wearable devices [23] are the so-called things. IoT, nowadays, finds applications across different domains like military and defense, healthcare, environmental monitoring, smart cities, and intelligent transportation systems [14]. Recently, one can find autonomous vehicles on the road communicating with each other and the roadside units. This emerging area is called Vehicular ADHOC Network (VANET) [4] from the security viewpoint, devices in IoT are vulnerable. These devices are prone to attacks since they have limited computing resources, and the communications are done over wireless media. Therefore, the security and privacy of the devices are of prime significance in IoT [34]. Furthermore, there are other inherent challenges within the IoT regime i.e., service discovery [10], network navigability [30, 33, 39], and trustworthiness [28, 32]. These challenges continue to become enormous, as the IoT scales up rapidly day by day.

Over the last decade, a novel approach with the potential to address these issues has been proposed by several researchers. That is, incorporating social networking concepts with IoT [8]. This method allows linked devices to create social connections with other devices in the network automatically and independently, imitating how people behave in social networks. The main driving force behind this strategy is the ability of socially "aware" IoT items to discover, choose, and compose services and information more organically. Therefore, in the paradigm of Social Internet of Things (SIoT), every device is represented as a node that is qualified of making social ties with other devices (nodes) autonomously following certain protocols or heuristics. In short, the objective of SIoT is to exploit the social relationships among devices in a vast network of IoT [20] thereby facilitating effective information discovery, promoting scalability, and enhancing communication between devices. As the devices are identified as friends (based on their social ties) trustworthiness among devices is reinforced [19].

The first and foremost task in SIoT is modeling a social network with a graph called the social graph. Then, it becomes easier to analyze and investigate the network parameters and their properties using graph theoretic approaches. The application of the graph theoretic approach to the SIoT network helps in gaining important insights regarding the importance or prominence of vertices/edges which can then be implemented with data visualization tools to provide intuitive and user-friendly interfaces for navigating and interacting with IoT data. At present graph-based approaches and graph-learning algorithms are gaining momentum in the machine learning research community. Graph-based modeling is applied in numerous fields like life science [40], community detection [22, 36], and prediction of new connections between communities [31].

A graph consists of a set of vertices connected by edges. Formally, it is represented as $G=(V, E)$, $V$ denotes the vertices set and E indicates the set of vertex pairs forming the edges, which are connected by an edge. The formulation of SIoT is done by the identification and classification of relationships between devices that exist in an IoT network. So far, the major classes or categories of [11] relationships identified are Co-Location Object Relationship (C-LOR), Ownership Object Relationship (OOR), Parental Object Relationship (POR), Co-Work Object Relationship (C-WOR), and Social Object Relationship (SOR). Nowadays one can find synergy between social networks and graph-based approaches in applications like the automatic detection of movement patterns of tourists [18] or the prediction of events based on the interaction of entities in social networks [12].

The rest of this work comprises the following sections: the literature survey is summarized in section 2. Section 3 defines methods used in social networks and SIoT. In section 4, the results are analyzed and discussed. The future work and conclusion are included in section 5.

## 2. Literature Review

The SIoT has emerged as an extension of the traditional IoT, integrating social relationships among smart objects to enhance trust, security, and service discovery. This section reviews key contributions in trust management, service discovery, and SIoT architectures, highlighting advancements, challenges, and future research directions.

Abdelghani *et al*. [1] suggested a Dynamic and Scalable multi-Level-Social Trust Model (DSL-STM) approach exclusively for SIoT contexts. Experiments on numerous simulated settings allow us to demonstrate the resilience and efficacy of DSL-STM. Mohammadi *et al*. [27] suggested a general reference model to reduce resource usage, and optimization decision theory was applied to optimal friend selection. The experimental outcomes revealed that selecting an adequate number of friends for each service exploration improves global navigability regarding average path length, degree distribution, and number of linkages.

Achir *et al*. [3] a thorough taxonomy of service discovery methodologies in the framework of IoT was evaluated based on many characteristics and criteria. The shortcomings and merits of every class in the taxonomy, as well as the context and requirements within which they can operate, are then examined. Alam *et al*. [5] explore trust management in the SIoT, focusing on architectures, relationships, and models for trust computation, aggregation, and updates. Trust attributes include social trust and quality of service, with feedback types classified as reputation-based, recommendation-based, and knowledge-based. Future directions emphasize privacy-preserving trust models to enhance security and reliability in SIoT environments.

Several architectures have been proposed and implemented for SIoT paradigm. Sociocast is one such initiative [9]. Sociocast is based on new primitive trusted group-oriented communication, and the in-network publish/ subscribe mechanisms. Additionally, it has flexible datacasting and both dynamic and selective firewalling. In all the major works reported so far on SIoT, the Santander city dataset [25] has been employed to test and validate the claims made. Any social network that satisfies the following three properties qualifies as a real-time network viz [37].

a) High average clustering coefficient.
b) Small world effect.
c) Scale free network.

In this work, the objective is to investigate whether SIoT network exhibits the aforesaid properties. The Santander city dataset has been chosen to validate both the local and global properties of a network degree distribution, node centrality, average degree, network diameter, clustering coefficient, modularity, average path length, network density, and giant component.

## 3. Methods

The concept of SIoT centers around making smart objects in an IoT network to collaborate and form social relationships with other objects similar to the way humans interact in social networks. The aim is to leverage the social connections between items inside an Internet of Things (IoT) context. to make efficient service discovery and composition which will promote network scalability and enhance communication between objects that are identified as friends [13].

This approach in turn would strengthen trustworthiness among objects. Graph theoretic measures can be applied to investigate various properties of such graphs. A graph is, as was previously said, a set of vertices joined by edges. In most cases, the names given to the vertices and edges vary depending on the domain. In computer science, vertices are called nodes and the edges are called links. In molecular physics, the vertices represent atoms and the edges are bonds. In social science, a graph represents the interaction (edges) between actors (nodes).

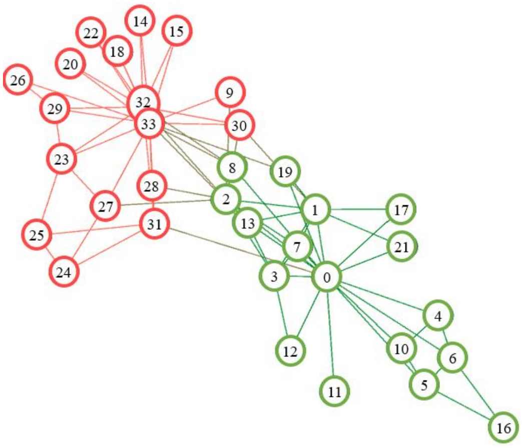



Figure 1. Karate club social graph.

A social graph is a network that represents the connection between an individual and another individual or item. For instance, the graph depicted in Figure 1 is a social network of the famous toy example- a karate club [38]. The network displays its 34 members as nodes and their interaction is represented as links between pairs of members. On the contrary, social networks namely Twitter, Facebook, and Instagram, are quite large, and interaction among the nodes is complex. The networks may be heterogeneous, i.e., a node may belong to one of several types. In the case of Twitter, one can attribute a node to a person or a tweet. In this work, the SIoT network has both homogenous and heterogeneous nodes.

Table 1. Vertices and edges in real world networks.

| Network | Vertex | Edge |
|---|---|---|
| WWW | Web pages | Hyperlinks |
| Biological neural network | Neurons | Synapses |
| Internet | Switch, Computer, Router | Unguided media, Guided |
| Power grid | Power stations, Sub-stations | Transmission lines |
| Air traffic control | Airports | Airway routes |
| Roadways | Cities | Roads |
| IoT network | Devices | Wireless connections |
| SIoT network | Devices | Social relationships |

Table 1 arrays many familiar networks (but not limited to) from our day-to-day experience and the respective interpretation of their vertices and edges.

As previously mentioned, SIoT stands for the convergence of technology from the IoT and social networking. This idea is fast gathering acceptance thanks to the key benefits it provides. These benefits are derived from the synergy of the social network concepts when applied to an IoT platform. The following are the benefits of SIoT:

1. Navigability in a dynamic network of trillions of items is made simpler.
2. Robustness when it comes to the management of trustworthiness among participating objects.

To summarize the various relationships of SIoT as discussed in [7]:

- POR: this relationship applies to items that are part of the same production batch, which are often homogeneous items produced by the same producer within the same time.
- C-LOR: applications where objects are used in the same place or site, including smart cities and homes, can make use of these linkages.
- C-WOR: this relationship encompasses those objects that work together to provide a common IoT service (e.g., security systems, early warning systems, intensive healthcare, etc.).
- OOR: a person/user may possess several devices and thus a relationship of type ownership is established among heterogeneous objects belonging to the same user.
- SOR: whenever people come in social contact (official or personal) with each other, their devices also sense these events and make instantaneous relationships. Table 1 arrays many familiar networks (but not limited to) from our day-to-day experience and the respective interpretation of their vertices and edges.

## 3.1. Graph Theoretic Properties

Graph theory deals with the study of graphs. A graph can be a completely connected graph where any pair of nodes is connected by an edge. Otherwise, it is known as a connected graph. If a graph contains two or more disjoint components, it is said to be a disjoint graph with many components. The edges can be either directed or undirected. Sometimes the edges will be assigned labels that represent the weight of the edge of the cost associated with that edge. A path is defined as a sequence of nodes that can be traversed following the edges between them. If a path exists from any node to itself, then such graphs are called cyclic graphs. Otherwise, the graph is called a Directed Acyclic Graph (DAG) which finds immense applications in computer science.

If a graph has as many edges as nodes, it is said to be a dense graph. Otherwise, the graph is called a sparse graph. Graphs can be implemented either utilizing an adjacency matrix or an adjacency list. A complete coverage of graph theory can be found in [24].

Big data analytics involves the usage of sophisticated methods from statistics to machine learning. Since in this paper, we are dealing with graph data structures, the graph theoretic approach is the apt method to analyze large networks. Social networks like Facebook, Twitter, etc., can be modeled as large networks or graphs. Some useful insights about the network can be deduced using graph analytical measures. Some of them include-clustering coefficients, scale-free networks, small world problems [2], community structure, resilience, etc., which have been successfully applied in several problems like the spread of epidemics, rumors, and fashion, the resilience of networks, and enhancing searching of network based on the concept of Homophily [29].

In this section, some of the graph properties are discussed. From the node point of view or in other words the local (node level) network structures. A node may possess in-degree and out-degree (directed graphs) are just degrees (undirected graphs). A node may be identified with properties like centrality which includes degree, eigenvector, Katz, PageRank, hubs and authorities, closeness, and betweenness. Sometimes, edge-centralities also play a significant role. For instance, we may be interested to know about the edge Betweenness centrality to identify bridges between two clusters of a network.

In social network analysis, many useful quantities are measured that capture interesting features of large social networks. These quantities or measures are called centrality measures. In a certain sense, centrality is a measure of a node's significance in a network.

#### 3.1.1. Degree Centrality

Knowing how important or central a node is in a network is captured by the degree of centrality. For example, in a terrorist network, it may be important to find the person who has a greater number of contacts with others. Equation (1) is utilized to calculate a node *i* degree centrality in graph *G*.

$$d_c = \frac{\#\ of\ edges\ of\ the\ node\ i}{\max_{|V|}(V)} \quad (1)$$

For example, for the sample graph shown in Figure 2, the degree centrality for nodes (1, 3, 5, 7) is (0.4, 1.0, 0.6, 0.2).

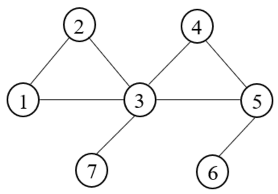


Figure 2. Sample undirected graph.

#### 3.1.2. Eigenvector Centrality

Sometimes, the importance of a node may not just be the degree of the node (as captured in degree centrality) but it may also depend on how important its neighbors are. This centrality measure is recursive where the significance of a node is determined by its neighbors which in turn depends on the neighbor's neighbors, and so on. Degree centrality treats all neighbors as equal and counts the number of neighbors a node has. But, in reality, all neighbors of a node are not equivalent. An important thing to note is that the eigenvector centrality of all vertices is either 0 or positive. It is expressed in Equation (2).

$$x_i' = \sum_j A_{ij}\ x_j \quad (2)$$

For instance, in the terrorist network, we may be interested to find the leader of the group. In such a situation a vertex with a high degree of centrality may not be the leader. On the other hand, a vertex that may have less but highly influential neighbors may be the leader.

#### 3.1.3. Katz Centrality

Katz centrality is a modified version of eigenvector centrality that addresses the problem of assigning centrality in a directed acyclic graph to a node without degrees and no in-degree, for example, a citation network. The problem with eigenvector centrality, when applied to DAG, is that nodes without in-degree are assigned zero centrality. Katz's centrality overcomes this problem by giving every vertex a small amount of centrality freely irrespective of its position in the network [35]. This can be denoted as in Equation (3).

$$x_i = \alpha \sum_j A_{ij}\ x_j + \beta \quad (3)$$

Where $\alpha$ and $\beta$ are positive constants. In Equation (3), the first term is the eigenvector centrality term and the second term is the freely awarded centrality to all the vertices. The eigenvector centrality term is weighted by the term $\alpha$ whose value is generally chosen as $\alpha$=1/*largest eigenvalue*.

### 3.2. Page Rank

In Katz centrality there is a problem of assigning higher centrality to a node pointed by high centrality node, i.e., suppose a node *i* with high centrality points to 100 other nodes. Then, each of the 100 nodes receive higher centrality because of node *i*. This should not be the case. Therefore, the centrality received from a node should be divided by the out-degree of that node. Thus, PageRank overcomes the problem of Katz centrality by using the Equation (4).

$$x_i = \alpha \sum_j A_{ij}\ \frac{x_j}{k_j^{out}} + \beta \quad (4)$$

### 3.3. Hubs and Authorities

In all the above centrality measures, a vertex was assigned a higher centrality if it was pointed to by other vertices with high centrality. But in certain networks, sometimes it is appropriate to assign a vertex with higher centrality if that vertex points to other vertices of high centrality. For example, in the case of citation networks, a review article cites many other articles and references which are important sources of subject knowledge. In this scenario, the important or authoritative papers have higher centrality in the network. Even though the review article itself may not have novelty, it helps researchers locate important information about a topic in the network. Thus, in such networks there are two important node types the authorities are the nodes that contain important information, and hubs are the nodes that take us to the location of the authorities. Figure 3 depicts a typical hub and authority network. This idea was proposed by Kleinberg [21]. When applying this idea to the World Wide Web (WWW), and is called Hyperlink Induced Topic Search (HITS). Mathematically this idea can be expressed as $x_i$ the authority and $y_i$ the hub in Equations (5) and (6).

$$x_i = \alpha \sum_j A_{ij}\ y_j \tag{5}$$

$$y_i = \beta \sum_j A_{ji}\ x_j \tag{6}$$

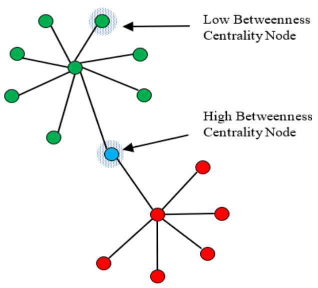



Figure 3. Hub and authority.

### 3.4. Closeness Centrality

Another important metric in social network is closeness centrality. The geodesic path is another name for the shortest path in a network that connects two vertices. The average shortest route between each vertex and a certain vertex is measured by the proximity centrality.

$$l_i = \frac{1}{n-1} \sum_{j(\neq i)} d_{ij} \tag{7}$$

The value of $l_i$ is less for vertices that are close to other vertices in the network. Hence such vertices will be able to propagate information to other vertices in a faster manner. Based on $l_i$, the closeness centrality is defined in Equation (8).

$$C_i = \frac{1}{l_i} = \frac{n}{\sum_j d_{ij}} \tag{8}$$

Revisiting the terrorist network, and closeness centrality will help us to find who in the network will spread information faster. The so-called "small world effect," another network-level feature, is strongly associated with closeness centrality. Regarding the Figure 2 graph the closeness centrality of nodes (3, 6) is (0.85, 0.42).

### 3.5. Betweenness Centrality

The number of times a node occurs on the shortest path between any two vertices in a network is known as betweenness centrality. This idea was proposed by Kleinberg [21], and Freeman [16]. For instance, let us assume that in a social network, there may be information or messages or news flowing between any pair of vertices. Now to count how many messages, on average, will have passed through each vertex en-route to their destination, we should consider vertices with higher count as high betweenness centrality vertices. Because of their ability to regulate the information that flows through them, vertices with high betweenness centrality will thus have a strong positive effect inside the network. Betweenness centrality can be expressed in Equation (9).

$$x_i = \sum_{s,t} n_{st}^i \tag{9}$$

Where $n_{st}^i$=1 if vertex $i$ lies on the path between $s$ and $t$ and $n_{st}^i$= 0, *otherwise*. $x_i$ gives the maximum value for the node $i$ that lies in the path between any source node $s$ and target node $t$. Thus, node $i$ has the highest betweenness centrality, we might claim. In our terrorist network, betweenness centrality will help us find the person who can spread messages as far as possible in the network. High and low betweenness centrality nodes are highlighted in Figure 4.

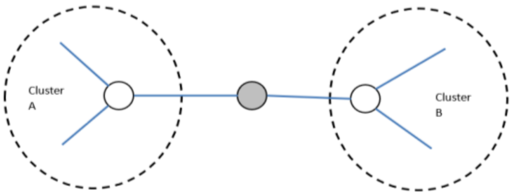



Figure 4. Betweenness centrality.

An important property to note in betweenness centrality is that a node may have high betweenness centrality in spite of having low degree. In Figure 5, the shaded node has low degree (=2) but its betweenness centrality is very high.

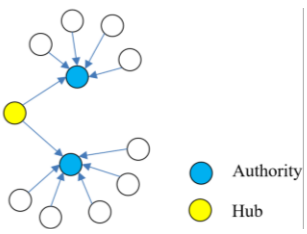



Figure 5. Betweenness vs. degree.

## 4. Experimental Results

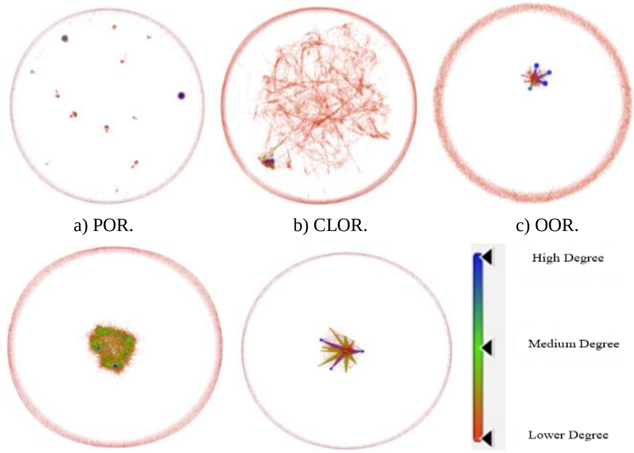
In the previous section, graph theoretic properties are summarized and it is proposed that the mathematical framework of graphs and their properties can be used to model complex systems. In the context of the Santander city SIoT dataset, graph theory can be used to analyze the relationships between the different sensors and the sensor network's general architecture. Many of the current research work in SIoT has been tested and validated on the Santander city dataset. Smart Santander is a unique city-level experimental dataset for validating applications and services of smart cities. Table 2 provides a detailed overview of the dataset.

Table 2. Dataset description.

| Dataset name | Santander city |
|---|---|
| Total number of nodes | 16, 216 |
| Total number of edges | 146, 117 |
| Graph type | Directed graph |
| Total number of device types | 16 (private-8 public-8) |
| Total number of private devices: | 14600 |
| List of private devices | Smart phone, Car, Smart fitness, Tablet, Smartwatch, Printers, PC, Home sensors. |
| Total number of public devices | 1616 |
| List of public devices | Transportation, Street light, Parking, Alarm, Indicators, Environment and Weather, POI, Garbage truck |
| Total number of brands | 12 |
| Total number of device models | 24 24 |

In this work, attempts have been made to represent the Santander city SIoT dataset as a graph treat each sensor as a node in the graph, and connect the nodes with edges that demonstrate the proximity between the sensors. The resulting graph is analyzed using various graph theoretic properties such as centrality measures, clustering coefficients, and community detection algorithms. Followed by visualization of the dataset has been done using the NetworkX [17] and Gephi.

The visualization of the Santander city dataset by using the Gephi tool is shown in Figure 6-a), b), c), d), and e). various insights can be gathered on various social IoT relationships. It can be observed that all SIOT relationships form a core-periphery graph structure. One or more larger-sized components may make up the core graph. Conversely, the periphery graph is made up of several smaller components. Figure 6-a) depicts the POR relationship in SIoT. The core component is highly fragmented. There are 12,987 components with majority of which are components of 2 to 3 nodes. The giant component in POR has a size of 677 nodes only which is 4.17% of the entire graph. In the CLOR relationship demonstrated in Figure 6-b), we can observe that the giant component in the core is well established constituting 51.27% of the entire network. OOR relationship is depicted in Figure 6-c) majority of the components lay in the periphery with a small core component constituting 8.99% of the entire graph. This is due to the devices under the same ownership are represented as cliques. Coming to the SOR relationship, there are two versions of the SOR dataset. As mentioned earlier in section 2, the SOR relationship is established between devices that momentarily come in close contact with each other when their owners meet. In SORv1 the graph depicts the SOR relationship between private mobile devices. Whereas in SORv2 the relationship is shown between public mobile devices. Both the relationships are depicted in Figure 6-d) and e) respectively. It can be noticed that the number of public mobile devices is less in the Santander dataset (the size of the core is smaller than that of the core in private mobile devices).




a) POR. b) CLOR. c) OOR.

d) SORv1. e) SORv2. f) Color scale.

Figure 6. Visualization of various SIoT relationships.

Figure 7 portrays a complete IoT network considering all social relationships. It is a connected graph with only one component comprising 16,216 nodes with 146,117 edges. This SIoT network follows the small world problem with an average path length of 4.22 and a network diameter of 8. In all the graphs the nodes are colored as per the legend shown. The nodes with higher degrees are colored in blue and those with lower degrees are colored in red. green nodes have a medium degree.

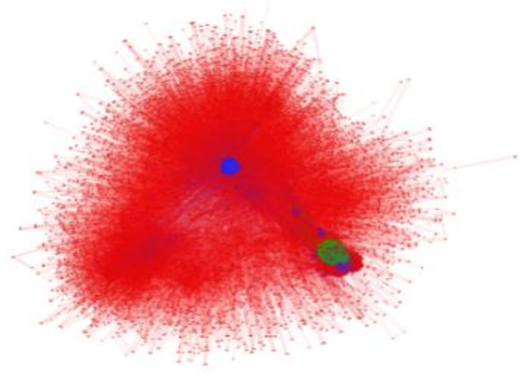


Figure 7. Visualization of Santander city SIoT dataset.

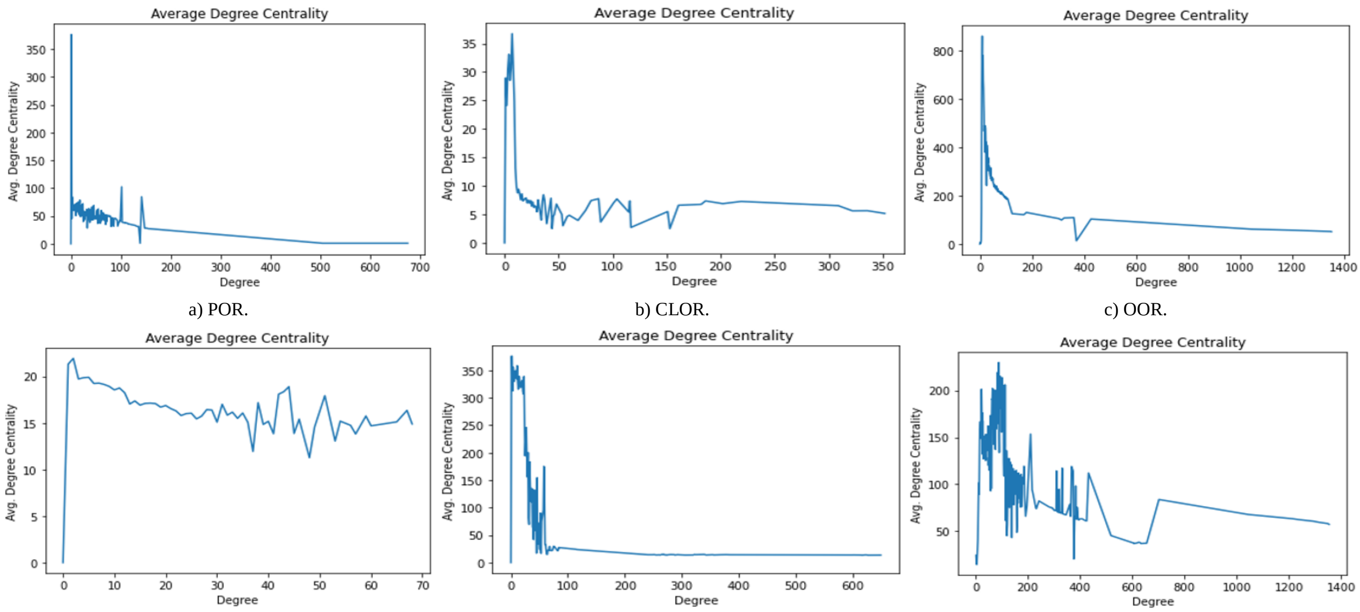
### 4.1. Performance Analysis

Since any social network can be effectively represented using graphs, investigating the Santander city SIoT dataset using a graph theoretic technique is the aim of this paper. The following metrics are analyzed and important inferences are drawn from the experiment.

#### 4.1.1. Degree Centrality

A graph's degree centrality indicates the degree to which nodes with higher degrees are central. Sometimes the nodes with very high degrees may occupy central positions in the graph. At other times it is also possible that nodes with high degrees may lay on the periphery of the network without having centrality like closeness and betweenness.

From Figure 8 it can be inferred that the social relationships namely POR, CLOR, OOR, and SORv2 exhibit almost similar characteristics-nodes with higher degrees do not occupy central positions in the network. Whereas SORv1 has more or less uniform average degree centrality for all degrees from 0 to 70. The same trend can be seen in the SIoT network shown in Figure 8-f) which follows more or less the trends in Figure 8-a), b), c) and e).



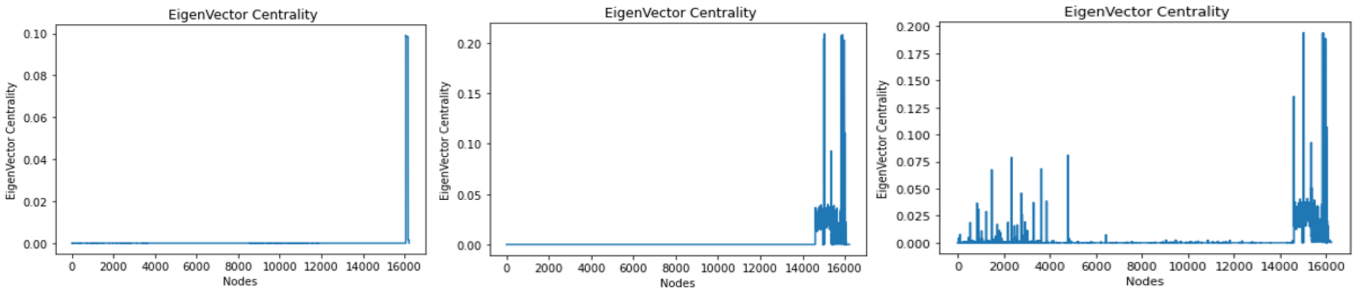

a) POR. b) CLOR. c) OOR.

d) SORv1. e) SORv2. f) SIOT.

Figure 8. Degree centrality of various SIoT relationships.

#### 4.1.2. Eigenvector Centrality

A node's eigenvector centrality has to be determined to calculate its effect in a graph. One global metric used to indicate a node's significance in a graph is called eigenvector centrality. However, eigenvector centrality is not meaningful in non-connected graphs. The largest(giant) component will dominate. Whereas the eigenvector centrality for all other components will be meaningless. This is quite evident from Figure 9. where eigenvector centrality cannot be measured for CLOR, SORv1, and SORv2. Other social networks have some giant component for whose node's eigenvector centrality can be computed.

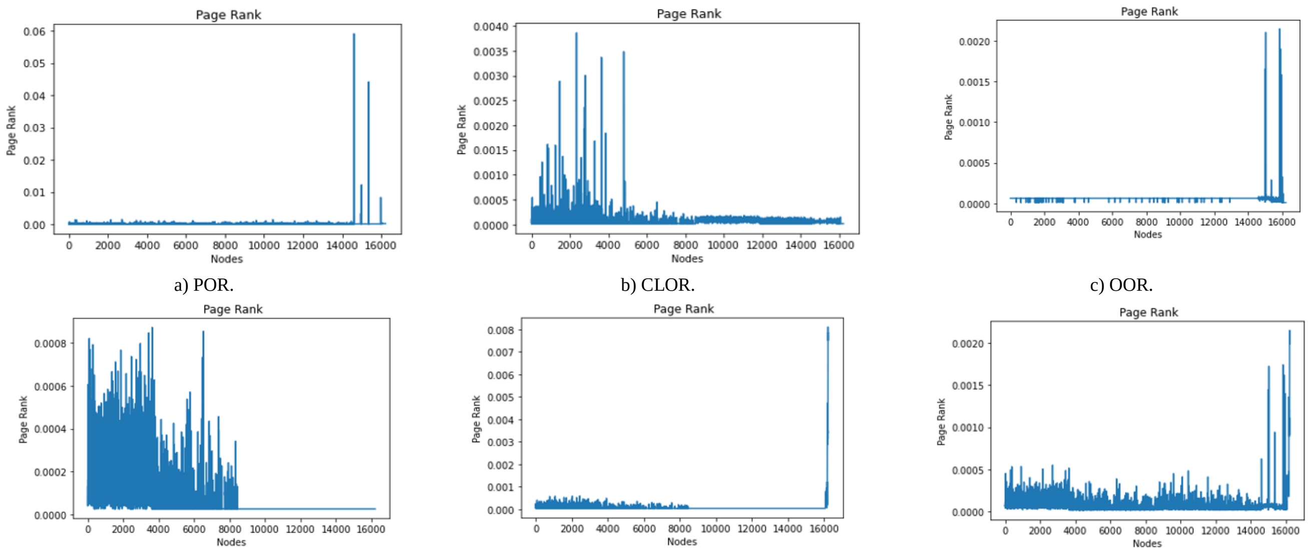



a) POR. b) CLOR. c) OOR.

Figure 9. Eigenvector centrality of various SIoT relationships.

#### 4.1.3. Page Rank

A node's relative importance in a graph is determined by its page rank. The importance of the given node does not depend on the degree of the node but it depends on the page rank of its neighbour nodes. A node may have fewer degrees but a high page rank if its neighbour has a higher page rank. Analyzing the page ranks of the nodes in different social relationships networks shown in Figure 10-a), b), c), d), e), and f), we can find similar patterns among CLOR, SORv1 and SORv2. Around 50% of the nodes have very little page rank owing to disjoint nodes and small components. Whereas in the case of POR, OOR, and SIoT some nodes are designated to have higher page rank compared to the rest of the nodes.



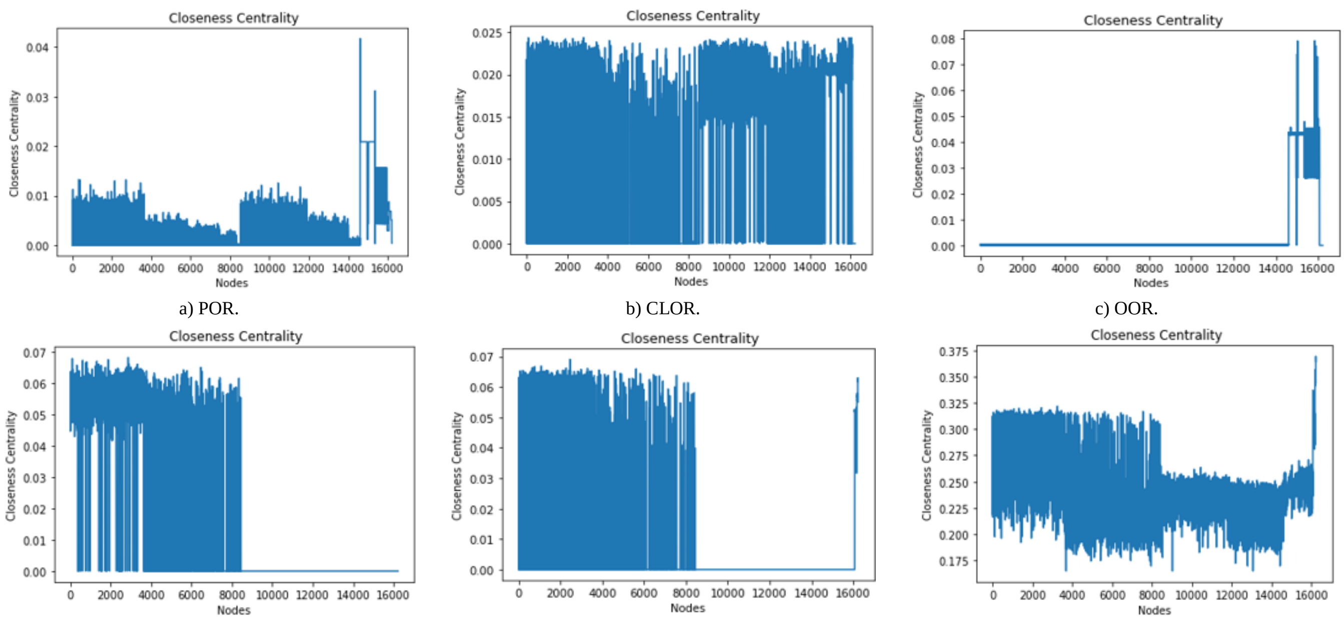

a) POR. b) CLOR. c) OOR.

d) SORv1. e) SORv2. f) SIOT.

Figure 10. Page rank of various SIoT relationships.

#### 4.1.4. Closeness Centrality

A node's closeness centrality indicates how near it is to every other node in a network. It is determined by the length of all the shortest paths between that node and every other node. A node with higher closeness centrality can quickly reach all other nodes, making it more central in the network. The same interpretation that applies to betweenness centrality also applies to closeness centrality. Since CLOR and SIoT networks have large connected components, the nodes exhibit closeness centrality uniformly. This can be inferred from Figure 11-b) and f). In other social relationships, we can observe that less than 50% of the nodes have almost negligible closeness centrality. This is a result of the graph's numerous solitary nodes and tiny linked elements. This is shown in Figure 11-d) and e). We can observe that in the SIoT network, almost all nodes exhibit significant closeness centrality from 0.175 to 0.375.



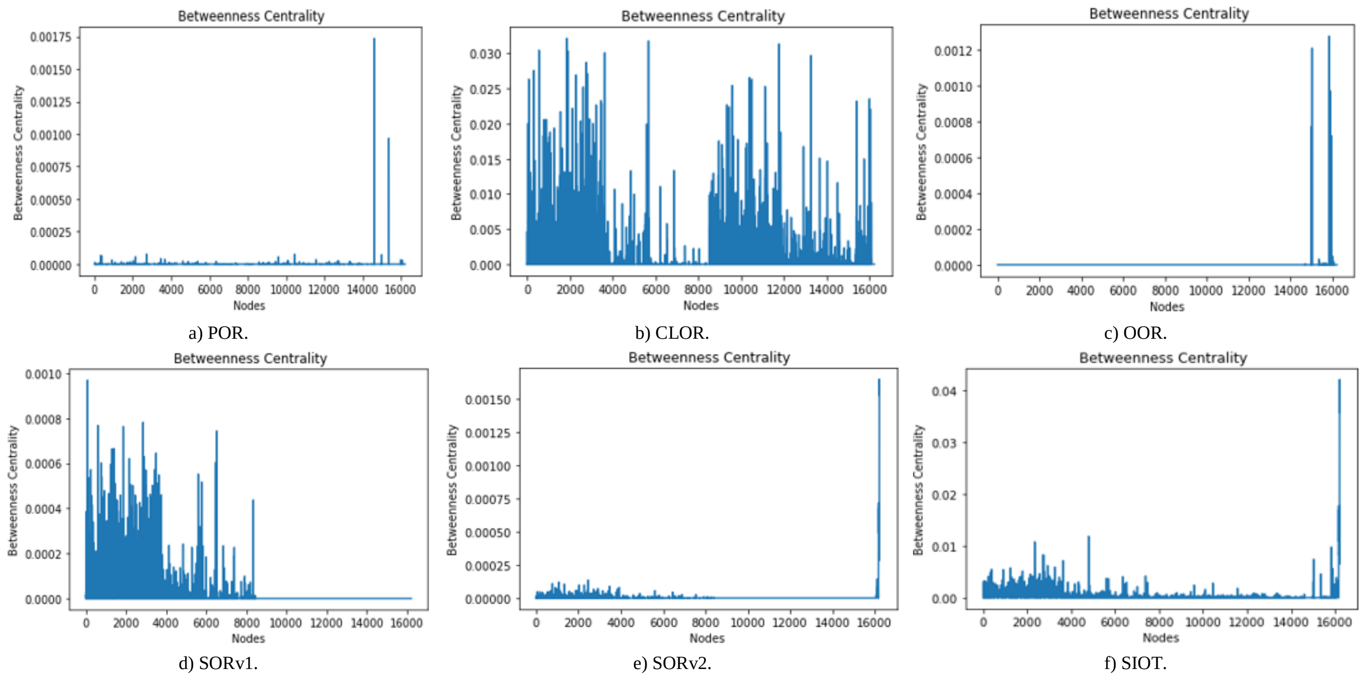

a) POR. b) CLOR. c) OOR.

d) SORv1. e) SORv2. f) SIOT.

Figure 11. Closeness centrality of various SIoT relationships.

#### 4.1.5. Betweenness Centrality

The term "betweenness centrality" describes a node's location between two or more clusters. Such a node attains higher betweenness centrality because the shortest path between nodes in two dissimilar clusters goes through that node. In this study, most of the SIoT relationship networks do not have significantly sized giant components except for CLOR. The SIoT network represents an IoT network that includes all the SIoT relationships and is composed of a single giant component. Hence betweenness centrality applies only two CLOR and SIoT as it is evident in the Figure 12-b) and f). Rest all other social relationships networks have many disjoint components.




a) POR. b) CLOR. c) OOR.

d) SORv1. e) SORv2. f) SIOT.

Figure 12. Betweenness centrality of various SIoT relationships.

### 4.1.6. Summary of Centrality Measures

Table 3 offers a thorough summary of the centralities associated with nodes in SIoT relationship graphs. Each type of SIoT relationship is associated with a specific centrality measure that best characterizes its structural properties. For instance, OOR is most accurately represented by degree centrality, highlighting the significance of node degree in this context. On the other hand, CLOR demonstrates a central tendency with betweenness centrality, showcasing the importance of node position in facilitating communication within clusters.

Table 3. Summary of various centralities in SIoT relationships.

| SIoT relationships | Degree centrality | | Betweenness centrality | | Eigen vector centrality | | Closeness centrality | | Page rank | |
|---|---|---|---|---|---|---|---|---|---|---|
| | Min | Max | Min | Max | Min | Max | Min | Max | Min | Max |
| **POR** | 0.0 | 0.0416 | 0.0 | 0.0017 | 6.8636e-26 | 0.0991 | 0.0 | 0.0416 | 2.8141e-05 | 0.0590 |
| **CLOR** | 0.0 | 0.0217 | 0.0 | 0.0321 | - | - | 0.0 | 0.0244 | 1.5713e-05 | 0.0038 |
| **OOR** | 0.0 | 0.0833 | 0.0 | 0.0012 | 3.0978e-36 | 0.2092 | 0.0 | 0.0790 | 9.3536e-06 | 0.0021 |
| **SORv1** | 0.0 | 0.0041 | 0.0 | 0.0009 | - | - | 0.0 | 0.0681 | 2.7162e-05 | 0.0008 |
| **SORv2** | 0.0 | 0.0400 | 0.0 | 0.0016 | - | - | 0.0 | 0.0690 | 3.3307e-05 | 0.0081 |
| **SIoT** | 6.1671e-05 | 0.0835 | 0.0 | 0.0420 | 1.2283e-10 | 0.1942 | 0.1650 | 0.3692 | 9.9942e-06 | 0.0021 |

Additionally, we can note the specific centrality characteristics of nodes in different SIoT relationships. Nodes in OOR relationships exhibit superior closeness centrality and eigenvector centrality, emphasizing their accessibility and influence. Meanwhile, POR relationships showcase nodes with higher page rank due to their hierarchical structure. In short, we can conclude that drawing attention to the dominance of closeness centrality in the SIoT dataset case study, suggests efficient navigability within the SIoT network. It is also emphasized that, through the strategic application of heuristics, nodes can locate any desired service with minimal hops, indicating a well-optimized and easily navigable SIoT environment.

Figure 13 illustrates the varied computational difficulty of several graph-theoretic techniques, including degree centrality (O(n)), closeness centrality (O(n²)), betweenness centrality (O(n³)), and optimized betweenness centrality (O (n log n)). The graph shows the increase in computing time as the number of nodes increases, with a distinct line for each approach. The log-log scale accurately depicts the exponential rise in complexity with network size. The graphic illustrates the significant difference in scaling between the techniques, with betweenness centrality seeing the most rise.

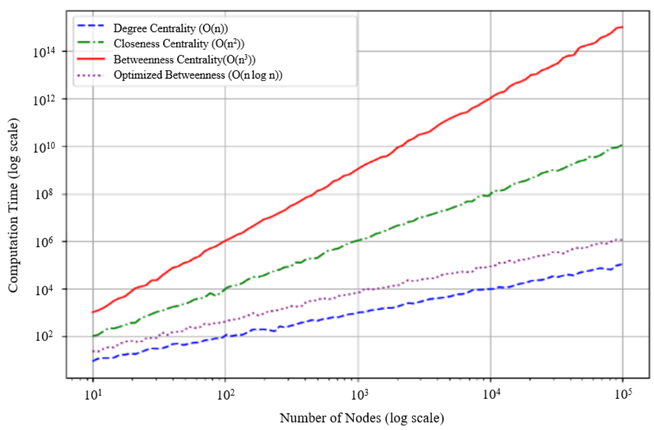



Figure 13. Computational complexity comparison.

In addition to local metrics, this study provides a foundation for incorporating global network properties such as betweenness centrality and clustering coefficient which enhances network robustness and efficiency.

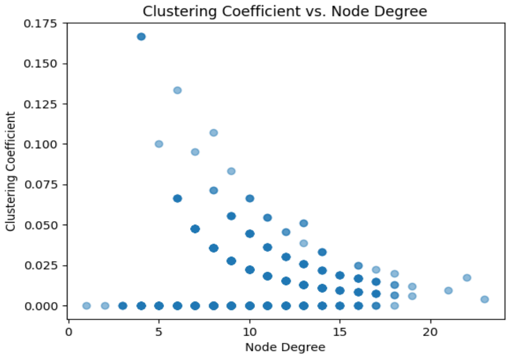



Figure 14. Clustering coefficient vs. node degree.

The clustering coefficient vs. degree in Figure 14 shows the node's connection and the possibility of creating close-knit clusters are related. Strong local connections between high-degree nodes are indicated by a larger clustering coefficient, which improves network dependability, trust, and communication efficiency. This finding is useful for both IoT service discovery and device collaboration in smart settings.

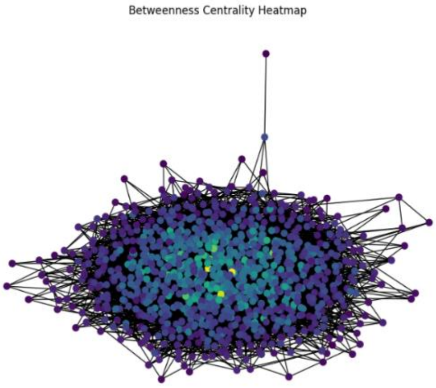



Figure 15. Betweenness centrality heatmap.

The betweenness centrality heatmap is shown in Figure 15 with nodes colored according to how much they help with communication. Higher betweenness centrality nodes serve as essential bridges facilitating effective routing and maximizing information transfer between various clusters. In SIoT applications, identifying these critical nodes helps with traffic optimization, network load balancing, and service allocation.

## 4.2. Discussion

This work aims to analyze SIoT network efficiency, service discovery, and trust management. A high degree of centrality in OOR achieves 43.68% which enhances personalized service recommendations and device-to-device communication, while betweenness centrality in CLOR achieves 85.6% which improves routing in smart city infrastructure. Closeness centrality in OOR (28.00%), SORv1 (24.14%), and SORv2 (24.45%) enable low-latency interactions which are essential for monitoring and emergency response. The SIoT network exhibits high closeness centrality across most nodes (ranging from 0.175 to 0.375), indicating efficient communication and low-latency service discovery, which enhances network navigability and responsiveness in IoT environments as shown in Figure 11. These findings confirm SIoT's small-world properties, supporting scalability, trust, and security by structuring device relationships. Strong local connections between high-degree nodes are indicated by a larger clustering coefficient, which improves network dependability, trust, and communication efficiency. This finding is useful for both IoT service discovery and device collaboration in smart settings as shown in Figure 14. Moreover, while the current model assumes static relationships, the framework can be extended to dynamic environments, allowing for real-time updates and adaptability to changing SIoT topologies. Time-evolving graph models can be integrated to analyze shifting centrality, connectivity, and service interactions, making the system more realistic and responsive. These insights contribute to network resilience, fault tolerance, and adaptive communication strategies.

## 5. Conclusions

In this study, we have employed Graph Theoretic Approaches to investigate the practical applications of SIoT within various social relationships, including POR, CLOR, CWOR, OOR, and SOR, using the Santander city dataset as a benchmark. They primarily focused on node-level parameters, particularly centrality measures. Notably, the OOR relationship displayed significant node degree (43.68%), outperforming other relationships in terms of degree centrality. Additionally, the CLOR relationship exhibited the highest betweenness centrality (85.6%), while closeness centrality demonstrated effectiveness in three relationships: OOR (28.00%), SORv1 (24.14%), and SORv2 (24.45%). Eigenvector centrality was most pronounced in OOR relationships (67.85%), whereas page rank measure was dominated by nodes in POR (79.94%). These findings address scalability, navigability, and trustworthiness issues in IoT networks by incorporating SORs. In conclusion, graph theory emerges as a potent framework for comprehensively analyzing the structure and attributes of the Santander city SIoT dataset, offering insights and revealing patterns that might otherwise remain concealed using alternative methods. Moreover, while the current model assumes static relationships, the framework is extended to dynamic environments in the future for real-time updates and adaptability to changing SIoT topologies. Time-evolving graph models are integrated to analyze shifting centrality, connectivity, and service interactions, making the system more realistic and responsive. In addition, future work will focus on creating scalable heuristics and approximation methods that keep analytical accuracy on massive real-world SIoT networks.

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## References

[1] Abdelghani W., Amous I., Zayani C., Sedes F., and Roman-Jimenez G., "Dynamic and Scalable Multi-Level Trust Management Model for Social Internet of Things," *The Journal of Supercomputing*, vol. 78, no. 6, pp. 8137-8193, 2022. https://doi.org/10.1007/s11227-021-04205-5

[2] Abdul R., Paul A., Junaid Gul M., Hong W., and Seo H., "Exploiting Small World Problems in a SIoT Environment," *Energies,* vol. 11, no. 8, pp. 1-18, 2018. https://doi.org/10.3390/en11082089

[3] Achir M., Abdelli A., Mokdad L., and Benothman J., "Service Discovery and Selection in IoT: A Survey and a Taxonomy," *Journal of Network and Computer Applications,* vol. 200, pp. 103331, 2022. https://doi.org/10.1016/j.jnca.2021.103331

[4] Al Shareeda M., Khalil A., and Fahs W., "Realistic Heterogeneous Genetic-based RSU Placement Solution for V2I Networks," *The International Arab Journal of Information Technology*, vol. 16, no. 3A, pp. 540-547, 2019. https://iajit.org/portal/PDF/Special%20Issue%20 2019,%20No.%203A/18600.pdf

[5] Alam S., Zardari S., Noor S., Ahmed S., and Mouratidis H., "Trust Management in Social Internet of Things (SIoT): A Survey," *IEEE Access*, vol. 10, pp. 108924-108954, 2022. DOI:10.1109/ACCESS.2022.3213699

[6] Al-Bzoor M., Ayyad W., and Alta'ani O., "A Survey on Efficient Routing Strategies for the Internet of Underwater Things (IoUT)," *International Journal of Electronics and Telecommunications*, vol. 68, no. 4, pp. 825-830, 2022. DOI:10.24425/ijet.2022.143891

[7] Ali S., Kibria M., Jarwar M., Lee H., and Chong I., "A Model of Socially Connected Web Objects for IoT Applications," *Wireless Communications and Mobile Computing*, vol. 2018, no. 1, pp. 1-20, 2018. https://doi.org/10.1155/2018/6309509

[8] Atzori L., Iera A., and Morabito G., "SIoT: Giving a Social Structure to the Internet of Things," *in IEEE Communications Letters*, vol. 15, no. 11, pp. 1193-1195, 2011. DOI:10.1109/LCOMM.2011.090911.111340

[9] Atzori L., Iera A., and Morabito G., "Sociocast: A New Network Primitive for IoT," *IEEE Communications Magazine*, vol. 57, no. 6, pp. 62-67, 2019. DOI:10.1109/MCOM.2019.1800917

[10] Atzori L., Iera A., and Morabito G., "The Internet of Things: A Survey," *Computer Networks,* vol. 54, no. 15, pp. 2787-2805, 2010. https://doi.org/10.1016/j.comnet.2010.05.010

[11] Atzori L., Iera A., Morabito G., and Nitti M., "The Social Internet of Things (SIoT)-When Social Networks Meet the Internet of Things: Concept, Architecture and Network Characterization," *Computer Networks*, vol. 56, no.16, pp. 3594-3608, 2012. https://doi.org/10.1016/j.comnet.2012.07.010

[12] Bok K., Kim I., Lim J., and Yoo J., "Efficient Graph-based Event Detection Scheme on Social Media," *Information Sciences*, vol. 646, pp. 119415, 2023. https://doi.org/10.1016/j.ins.2023.119415

[13] Chen Y., Tao Y., Zheng Z., and Chen D., "Graph-Based Service Recommendation in Social Internet of Things," *International Journal of Distributed Sensor Networks*, vol. 17, no. 4, pp. 1-13, 2021. https://doi.org/10.1177/15501477211009047

[14] Devaki M., Sathiamoorthy J., and Usha M., "IoT Based Air Quality Monitoring Using Densenet in Urban Areas," *International Journal of Data Science and Artificial Intelligence*, vol. 2, no. 4, pp. 121-127, 2024. https://www.scribd.com/document/767256623/IOT-BASED-AIR-QUALITY-MONITORING-USING-DENSENET-IN-URBAN-AREAS

[15] Drozd W. and Kowalik M., "Application of the Internet of Things (IoT) Technology for Controlling Air Purification in the Apartment," *Archives of Civil Engineering*, vol. 69, no. 2, pp. 41-52, 2023. https://journals.pan.pl/dlibra/publication/145251/edition/127736/content

[16] Freeman L., "Centrality in Social Networks: Conceptual Clarification. Social Network: Critical Concepts in Sociology," *Londres: Routledge*, vol. 1, no. 3, pp. 238-263, 1979. https://doi.org/10.1016/0378-8733(78)90021-7

[17] Hagberg A., Swart P., and Schult D., "Exploring Network Structure, Dynamics, and Function Using NetworkX," *in Proceedings of the 7th Python in Science Conference*, Los Alamos, pp. 11-15, 2008. https://www.osti.gov/biblio/960616

[18] Hu F., Li Z., Yang C., and Jiang Y., "A Graph-based Approach to Detecting Tourist Movement Patterns Using Social Media Data," *Cartography and Geographic Information Science*, vol. 46, no. 4, pp. 368-382, 2019. https://doi.org/10.1080/15230406.2018.1496036

[19] Jiang W., Wang G., Bhuiyan M., and Wu J.,

"Understanding Graph-based Trust Evaluation in Online Social Networks: Methodologies and Challenges," *ACM Computing Surveys*, vol. 49, no. 1, pp. 1-35, 2016. https://doi.org/10.1145/290615

[20] Joshua K. and Prabhu A., "Efficient Data Search and Retrieval in Cloud Assisted IoT Environment," *International Journal of Data Science and Artificial Intelligence*, vol. 2, no. 1, pp. 1-6, 2024. https://kitspress.com/journals/IJDSAI/content/02/01/IJDSAI_V02_01_01_06_25112024.pdf

[21] Kleinberg J., "Authoritative Sources in a Hyperlinked Environment," *Journal of the ACM*, vol. 46, no. 5, pp. 604-632, 1999. https://doi.org/10.1145/324133.324140

[22] Madavarapu J., Hasibul I., Appathurai A., Safdar G., Muthukumaran N., and Gnanamalar J., "Heterogeneous Energy Harvesting Techniques for Smart Home IoT Acceleration," *IEEE Access*, vol. 12, pp. 73667-73675, 2024. DOI:10.1109/ACCESS.2024.3397664

[23] Madavarapu J., Nachiyappan S., Rajarajeswari S., Anusha N., Venkatachalam N., Madavarapu R., and Ahilan A., "Hot Watch: IoT based Wearable Health Monitoring System," *IEEE Sensors Journal*, vol. 24, no. 20, pp. 33252-33259, 2024. DOI:10.1109/JSEN.2024.3424348

[24] Malik N., "Graph Theory with Applications to Engineering and Computer Science," *Proceedings of IEEE*, vol. 63, no. 10, pp. 1533-1534, 1975. DOI:10.1109/PROC.1975.9996

[25] Marche C., Atzori L., Pilloni V., and Nitti M., "How to Exploit the Social Internet of Things: Query Generation Model and Device Profiles' Dataset," *Computer Networks*, vol. 174, pp. 107248, 2020. https://doi.org/10.1016/j.comnet.2020.107248

[26] Maris Murugan T., and Jeyam A., "IOT-Enabled Protein Structure Classification via CSA-PSO Based CD4.5 Classifier," *International Journal of Data Science and Artificial Intelligence,* vol. 1, no. 2, pp. 41-51, 2023. https://www.scribd.com/document/767253749/IOT-ENABLED-PROTEIN-STRUCTURE-CLASSIFICATION-VIA-CSA-PSO-BASED-CD4-5-CLASSIFIER

[27] Mohammadi V., Rahmani A., Darwesh A., and Sahafi A., "Trust-Based Friend Selection Algorithm for Navigability in Social Internet of Things," *Knowledge-Based Systems*, vol. 232, pp. 107479, 2021. https://doi.org/10.1016/j.knosys.2021.107479

[28] Nagamalla V., Kumar B., Janu N., Preetham A., Gangadharan S., Alqahtani M., and Ratna R., "Detection of Adulteration in Food Using Recurrent Neural Network with Internet of Things," *Journal of Food Quality*, vol. 2022, pp. 1-11, 2022. https://doi.org/10.1155/2022/6163649

[29] Newmann M., *Networks: An Introduction*, Oxford University Press, 2010. https://doi.org/10.1093/acprof:oso/9780199206650.001.0001

[30] Nitti M., Atzori L., and Cvijikj I., "Friendship Selection in the Social Internet of Things: Challenges and Possible Strategies," *IEEE Internet of Things Journal*, vol. 2, no. 3, pp. 240-247, 2015. DOI:10.1109/JIOT.2014.2384734

[31] Palagan A., Gupta S., Dhas A., Taware S., Ravi Chakravarthi R., and Ahilan A., "A Wireless Body Area Sensor Based IoT Scheme for Healthcare Applications," *Research Square*, pp. 1-16, 2021. https://doi.org/10.21203/rs.3.rs-891137/v1

[32] Prasad M. and Reddy D., "Light Weight Clustered Trust Sensing Mechanism for Internet of Things Network," *International Journal of Electronics and Telecommunications*, vol. 68, no. 1, pp. 5-12, 2022. https://doi.org/10.1080/03772063.2022.2130449

[33] Ravindran V., "IoT Based Malware Attack Detection using Layered Deep Learning Framework," *International Journal of Computer and Engineering Optimization*, vol. 1, no. 2, pp. 43-49, 2023. https://kitspress.com/journals/IJCEO/index.php?info=15&manuscriptid=67

[34] Selvaraj S., Thangarajan R., and Saravanan M., "Trust-Based and Optimized RPL Routing in Social Internet of Things Network," *in Proceedings of the Mobile Computing and Sustainable Informatics Conference*, Patan, pp. 513-529, 2022. https://doi.org/10.1007/978-981-16-1866-6_36

[35] Sharkey K., "A Control Analysis Perspective on Katz Centrality," *Scientific Reports*, vol. 7, no. 1, pp. 1-8, 2017. https://doi.org/10.1038/s41598-017-15426-1

[36] Wang T., Tan J., Ding W., Zhang Y., Yang F., Song J., and Han Z., "Intercommunity Detection Scheme for Social Internet of Things: Compressive Sensing over Graphs Approach," *IEEE Internet of Things Journal*, vol. 5, no. 6, pp. 4550-4557, 2018. DOI:10.1109/JIOT.2018.2837048

[37] Yan C., "Network Model with Scale-Free, High Clustering Coefficients, and Small-World Properties," *Journal of Applied Mathematics*, vol. 2023, no. 1, pp. 1-11, 2023. https://doi.org/10.1155/2023/5533260

[38] Zachary W., "An Information Flow Model for Conflict and Fission in Small Groups," *Journal of Anthropological Research*, vol. 33, no. 4, pp. 452-473, 1977. DOI:10.1086/jar.33.4.3629752

[39] Zarzoor A., "Enhancing IoT Performance via Using Mobility Aware for Dynamic RPL Routing Protocol Technique (MA-RPL)," *International*

*Journal of Electronics and Telecommunications*, vol. 68, no. 2, pp. 187-191, 2022. DOI:10.24425/ijet.2022.139866

[40] Zhu S., Yu T., Xu T., Chen H., et al., "Intelligent Computing: The Latest Advances, Challenges, and Future," *Intelligent Computing*, vol. 2, pp. 1-45, 2023. DOI:10.34133/icomputing.0006

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