Fuzzy Inference Modeling Methodology for the Simulation of Population Growth

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Abstract: *This paper presents the use of fuzzy inference to provide a viable modeling and simulation methodology for the estimation of population growth in any country or region. The study is motivated by the classical complex and time-consuming growth modeling and prediction methods. The related design issues are presented and the fuzzy inference model for population growth is derived. The human social and economic factors which affect the growth and which underly the parameters used in the classical population projection methods are fuzzified. They are then used as inputs to a fuzzy population growth model based on fuzzy inferences so as the population growth rate is evaluated. The fuzzy population model is simulated using an existing CAD tool for fuzzy inference which has been developed and described elsewhere by the authors. The results obtained using different existing defuzzification strategies and a recently introduced one are compared with the actual population growth rates in some countries.*

Keywords: *Fuzzy inference, modeling, simulation, population growth, defuzzification.*

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1. Introduction

Coale and Trussell [3] noted the virtual absence of the development and steep decline in the use of demographic models during the past decade, with the exception of the use of demographic simulation models to evaluate demographic estimation techniques and the use of model schedules in population projections. The ability to estimate interstate transition rates or (probabilities) from population distributions has many potential applications in demography. Iterative Proportional Fitting (IPF) and Relative State Attraction (RSA) have been used for such estimation [11].

The theory of fuzzy sets [19] and the resulting fuzzy logic [18] have been regarded as highly valuable tools that can be used to simplify and enhance the analysis and design of complex humanistic systems and processes by employing the principles of approximate reasoning [20, 21]. Fuzzy logic has been successfully applied in many areas in science and engineering. Of particular importance is the application of fuzzy logic in the modeling and simulation of non-linear ill-defined and complex systems which are too difficult to model using classical methods [2, 6-9, 22].

The application of fuzzy logic relies on the fuzzification of the input variables that affect the output of the system to be modelled, the fuzzification of the output variable of the system, and the relationship between the input and output variables through a set of fuzzy or linguistic inference rules expressed in the form of IF-THEN rules. Also, of importance in the design of a fuzzy system is the selection of the defuzzification strategy that is to be applied to the fuzzy output obtained as a response to a particular crisp input.

In this paper the principles of fuzzy inference are used to provide a general fuzzy population growth model that can apply to estimate the growth in any country or region. The introduction of the fuzzy model has been motivated by the complex and time-consuming classical and stochastic demographic models that are usually applied in the prediction of population growth [3]. It is also motivated by the fact that the fuzzy model allows for the exploitation of what is described by Zadeh as "tolerance for imprecision" [16, 20]. This is of utmost importance if we consider the approximate nature of the values assumed by the variables or factors that mainly affect the population growth rate and the human-related nature of these factora and their relationship to the growth in population (see sections 5 and 6).

In giving a fuzzy model to the problem concerned with the determination of the population growth rate, the human social and economic factors such as female education, average income, etc. that affect the growth and which underly the parameters used in the classical population growth projection methods are fuzzified. Fuzzification is also attached to the growth itself regarded as the output of the model. A set of inference rules is used to provide the relationship between the fuzzy inputs and outputs of the model. The crisp population growth rates are determined by applying defuzzification strategies. Ultimately, the rates obtained through the use of the fuzzy model and an existing CAD tool specially prepared for the simulation of such models are compared with the actual rates in a number of countries.

Before we provide the detailed description of the fuzzy population model (sections 5 and 6), some background material in fuzzy logic inference is offered in section 2. Section 3 gives an insight on the different defuzzification strategies that are normally utilized in fuzzy methodology. This is in addition to a new defuzzification strategy which has recently been introduced by the authors [10]. Section 4 emphasizes on the classical methods of population projection. Sections 5 and 6 provide a detailed analysis and justification of the fuzzy population growth model and its underlying factors. The results of the fuzzy model, expressed as population growth rates, are presented and discussed in section 7. Conclusive comments are offered in section 8.

2. Background in Fuzzy Logic Inference

Fuzzy logic provides a unique and effective way to draw conclusions (system outputs) from vague and imprecise information (system inputs). Fuzzy logic is based on the concept of a *fuzzy set* which is characterized by a *membership function* taking values in the interval (0, 1) in place of 0 or 1 only as in *crisp sets*. Fuzzy statements, which form the backbone of fuzzy logic, are not, therefore, either true or false. Instead, they assume intermediate truth values represented by *membership grades*.

A fuzzy inference system contains a fuzzifier, a defuzzifier and a set of inference rules. Fuzzification consists of assigning a number of fuzzy sets that describe the different fuzzy states of the system input and output variables. Defuzzification consists of converting each fuzzy output that is obtained for a particular crisp input and as a result of implementing the inference rules into a crisp output so that it can be used for practical purposes. The inference rules, usually expressed in the form of IF-THEN rules, provide the necessary connection between the system input and output fuzzy sets.

In order to design a fuzzy inference system, the first step is to specify the input and output variables of that system. The second step is to associate with each variable different membership functions deemed suitable by the system designer. In the third step, the designer needs to define the rules so as to provide a suitable relationship between input and output fuzzy sets. Finally, a defuzzification strategy has to be selected to yield the crisp output(s).

A collection of N inference rules for a system with two input variables; x and y and one output variable; z, and whose form is typical in fuzzy systems is as follows [9]:

$$
\begin{aligned}
&R_1\text{: IF } x \text{ is } A_1 \text{ AND } y \text{ is } B_1\text{, THEN } z \text{ is } C_1\\
&R_2\text{: IF } x \text{ is } A_2 \text{ AND } y \text{ is } B_2\text{, THEN } z \text{ is } C_2\\
&R_3\text{: IF } x \text{ is } A_3 \text{ AND } y \text{ is } B_3\text{, THEN } z \text{ is } C_3\\
&R_j\text{: IF } x \text{ is } A_j \text{ AND } y \text{ is } B_j\text{, THEN } z \text{ is } C_j\\
&R_N\text{: IF } x \text{ is } A_N \text{ AND } y \text{ is } B_N\text{, THEN } z \text{ is } C_N \qquad (1)
\end{aligned}
$$

Of course, more than two input variables can be considered and the rules can be rewritten accordingly. $A_1, \ldots, A_N$ and $B_1, \ldots, B_N$ are the linguistic or fuzzy values that could be taken respectively by the input variables x and y. $C_1,\ldots,C_N$ are the fuzzy values that could be assumed by the output variable z. The implementation of the inference rules to obtain a fuzzy output for a crisp input (x_0, y_0) involves the use of the fuzzy logic operators, such as AND, OR, etc. and their mathematical representation; such as minimum, maximum, etc. It also involves the use of Zadeh's compositional rule of inference [20]. Actually, the inference rules in (1) can be represented by a fuzzy relation as follows [9]:

$$R = [(A_1 \cap B_1)\ X\ C_1] \cup [(A_2 \cap B_2)\ X\ C_2] \cup \ldots \cup [(A_N \cap B_N)\ X\ C_N] = \bigcup_{j=1}^{N} [(A_j \cap B_j)\ X\ C_j] \qquad (2)$$

In this relation, the symbol $\cup$ is taken as a representation of the OR operator introduced between the rules and $\cap$ accounts for AND, which is used in the antecedent parts of the rules. Symbol X represents fuzzy implication or THEN operator. The fuzzy output that corresponds to a crisp input pair (x_0, y_0) is given by

$$C(z) = R(x_0, y_0, z) \qquad (3)$$

If the AND, OR and THEN operators are replaced respectively by minimum, maximum, and minimum, then equation (3) with R as in (2) can be expressed as:

$$C(z) = \max_{1 \le j \le N} [A_j(x_0) \wedge B_j(y_0) \wedge C_j(z)] \qquad (4)$$

In their general form, the inference rules should satisfy two reasonable properties [9]:

1. Completeness: This means that the system can generate an output for any input fuzzy state.
2. Consistency: The inference rules are inconsistent when we have two or more rules with almost the same condition parts and highly different fuzzy sets are assigned to the consequents of these rules. Failure to observe consistency in the inference rules leads to unsatisfactory results.

3. Defuzzification Strategies

As noted in the previous section, in order to make the output of a fuzzy system available for practical use, it has to be crisp or non-fuzzy. Thus, a defuzzification strategy needs to be implemented. There are several defuzzification techniques, from which the designer could choose and apply the one that is most appropriate for a specific application and assignment of membership functions and rules. Defuzzification can be applied to the fuzzy output, as expressed in equations (3) or (4). In such a case, the First Of Maxima (FOM), Center Of Gravity (COG), and Center Of Sums (COS) methods can be used [5, 9]. It can also be directly applied from within the implementation of the inference rules and without passing by the fuzzy output first and then defuzzifying it. The Min-Max Weighted Average Formula (Min-Max WAF) [1, 9] or the Product Average Formula (PAF), introduced by the authors in [10], can be used in this case. These formulas apply when the output states are crisp or when a crisp representation is adopted for the fuzzy output states. It is worth mentioning in this instance that the PAF method is an improved version of the min-max WAF and it is capable of providing a smooth input-output characteristic of the fuzzy system as compared to the characteristic obtained using the min-max WAF [10]. Furthermore, the FOM applies to the output fuzzy set C (z) as in equations (3) or (4) by taking as a defuzzified value the smallest z at which the maximum membership degree in C (z) is achieved. The COG method applies to C (z) as:

$$COG\ [C(z)] = \int_{-\infty}^{\infty} zC(z)dz \Big/ \int_{-\infty}^{\infty} C(z)dz \qquad (5)$$

The discrete version of (5), which is useful in computer applications is:

$$COG\ [C(z)] = \sum_{i=1}^{n} z_i C(z_i) \Big/ \sum_{i=1}^{n} C(z_i) \qquad (6)$$

where n is the number of sample values (sample size) within the range of the output variable.

The COS uses the forms of the formulas in equations (5) and (6) but it applies them to the sum of the membership functions of the output fuzzy sets each of which is obtained as a result of the application of a collection of rules having the same fuzzy consequent.

4. Classical Population Projections

During the period of time extending between 1970 and 1988, intensification of the slowdown in the world demographic growth took place owing to the fall in the fertility that reached its maximum in the second half of the 1960's both in the developed and in the third world countries as a whole [15]. The rate of increase of the world population was 1.86% for 1955-1960 and 2.04% for 1965-1970. But it fell to 1.97% for 1970-1975, 1.75% for 1975-1980 and 1.67% for 1980-1988. The figure of 2.04% is historic, for it represents the absolute peak of the world population growth rate and it is anticipated that it will never be seen again. The main method used to project the growth in population through the estimation of population growth rates of countries is the component method [3, 13].

4.1. The Component Method

This method involves the use of the separate projections of mortality, fertility, immigration and emigration which are applied by age-sex groups. The total population growth is then obtained by combining the projections for the age-sex groups. The component method is considered complex since it relies on the use of complicated stochastic models to estimate future figures of the above-mentioned components that are regarded as the major factors that affect the population growth [3]. These estimates are usually obtained through the use of probabilities related to the likelihood of human behavior or acts. That is, a probability of marrying, giving birth, migrating, or dying within a given interval of future time is attached to each age-sex group of the population. Because the error associated with the projection of each component may be rather large, the component method is not considered a highly accurate one, but still employed in population projections [13].

As already mentioned, the computations in the component method are carried out separately for age-sex groups and on the basis of separate allowances for components. Specifically, one starts with the population distributed by age and sex at the base, applies assumed survival rates and age-sex-specific fertility rate or birth probabilities, and makes allowances for net migration. The base population should be the latest dependable estimates of the national population distributed by age and sex. The age groups usually have five-year class intervals. Five-year life-table survival rates can be applied to this base population, to bring it forward five years at a time and to allow for deaths in the interval. Births are usually computed by applying five-year age-specific fertility rates to the women of childbearing age at the middle of each five-year time interval. In any case, the determination of the prospective changes in fertility and mortality involves the use of some complex formulae.

Furthermore, the results obtained by the component method are not highly accurate, as previously stated, and the level of accuracy depends on the available statistical data pertaining to every age-sex group. Also, the calculations involved in this method are immense and time consuming. A computer simulating population projections, using such methods, was devised under the supervision of the UNESCO which requires the availability of the following parameters [14]: Population total, males and females, dependency ratio, child/ women ratio, median age, sex ratio, average annual rates of growth, rate of national increase, crude birth rate, crude death rate, crude net migration rate, estimated number of birth, estimated number of death, estimated number of migrants, gross reproduction rate, net reproduction rate, total fertility rate, general fertility rate, mean age of child-bearing, expectation of life at birth, and Infant mortality rate.

Henceforth, due to the complexity and lack of highly accurate population growth figures that result from the application of the classical methods in the projections of population growth, an alternative simpler method for estimating population growth that uses the theory of fuzzy sets and fuzzy logic inference is proposed and described in sections 5 and 6. Since fuzzy logic is based on the application of the "approximate reasoning" principles, it seems to provide a good solution for the modeling and simulation of population growth. The development of the fuzzy model involves identifying the factors that underly the parameters used in the classical methods, affect the population growth rate and are fit to be modeled using fuzzy sets and fuzzy inference.

5. Population Growth Factors

The effects of several factors on the growth rate of population in many regions around the world was studied. Economic status, religion, education, occupation, rural-urban factors, and marriage patterns are mainly the factors that influence the increase in population. The above stated factors were reduced to only four important ones directly affecting the population growth [12]:

- Economic status: Represented by the female average income per month.
- Education: Represented by the percentage of educated females in a certain region.
- Female employment: Represented by the percentage of female participation in social activities other than household.
- Female marriage age: Represented by the average age at which female marry in a certain region.

In what follows emphasis is placed on each of these four factors and the way they relate to the population growth rate. By considering the fuzzy logic inference model as described in section 2, the above-noted four factors, denoted by INCOME, EDUC, PART, and AGE are assumed as the input variables of a fuzzy system whose output variable is the population growth rate, denoted by POP. Different fuzzy sets are attached to each of the input variables and to the output variable. The input and output fuzzy sets are related by a set of IF-THEN inference rules that satisfy the basic properties of completeness and consistency. Also, the ranges of these sets and the input and output variables are taken so as to accommodate to the best possible, universally accepted standards as they relate to what is considered, low, medium or high female income, marriage age, etc. (section 6).

In addition to the discussion offered below on the effect of the chosen inputs on the population growth output (sections 5.1.-5.4.), it is worthwhile to mention at this point the interrelationship between the above four chosen inputs and its implications on the inference rules design. Actually, the setting of the 81 inference rules has been done by accounting for this interrelationship or correlation (i. e., the way the states of the input variables agree with each other) and also the collected data pertaining to several countries [12, 17]. Consider the following inference rule:

IF INCOME IS LOW AND EDUC IS LOW AND PART IS LOW AND AGE IS LOW THEN POP IS VHI (7)

It is self-evident that if the EDUC input is low then this will influence the INCOME to be low, and if the AGE input is low, the PART input is expected to be low. Of course, that does not necessarily mean that other factors may not affect the INCOME input in an upward trend despite the fact that the EDUC input may be low. This is obviously a function of the social fabric setup of the region or country under study. The following inference rule accounts for such a possibility.

IF INCOME IS HI AND EDUC IS LOW AND PART IS HI AND AGE IS HI THEN POP IS LOW (8)

It can also be seen from (8) that a high AGE input affects the PART at an upward trend (i. e., HI). However, it is possible for other factors to affect the PART and cause it to be medium although AGE might be high. The following inference rule considers this alternative situation.

IF INCOME IS HI AND EDUC IS LOW AND PART IS MED AND AGE IS HI THEN POP IS MED (9)

When comparing (8) and (9) it can be seen that the reduction in the PART input value has caused the POP output to increase (see section 5.3).

5.1. Economic Status

A well-established fact is that poor people have more children than the rich. A study of the direct relation between fertility and income was conducted among female office employees in public services. This study showed that the average number of children was *highest* for those with the *lowest* incomes and decreased with rising income. Hence, it can be concluded that the population growth rate is inversely proportional to the female income per month [12]. The INCOME input factor is taken to account for the economic status in the fuzzy population model.

5.2. Education

Studies of fertility in relation to the education of the wife have generally shown that the *higher* the grade attained in school, the *fewer* the number of children.

The low fertility of college or university graduates had been verified. The education factor is accounted for in the fuzzy population model as the percentage of educated females in a certain region abbreviated as the EDUC input factor. Collected data showed a variation between 0%-100% for the percentage of educated females in different regions of the world [12].

5.3. Employment of Women

Employment of women, or the participation of women in social activities other than household, is also an important factor affecting the rate of population growth. Several censuses have shown that married women with full-time employment throughout their marriage lives had far *fewer* children and a *much higher* incidence of childlessness than those who had never been employed [12]. For example, a study showed that the average number of children born by women who had been married ten to fourteen years was around 1.2 for those who were *heavily employed*, and 2.2 for other women [12]. The fertility of wives with intermediate employment histories ranked between these extremes, varying, in general, inversely with the extent of the employment. The female employment is represented in the fuzzy population model by the PART input factor and it varies between 0%-50%, since it never exceeded this range.

5.4. Female Marriage Age

Available data show that in countries of *high fertility* (developing countries) marriage of women, and as a result child bearing, occurs at an early age. Thus, the *earlier* the marriage age, the *higher* the number of delivered children. This phenomenon is nearly universal. For example, in Jamaica, studies show that 48% of women aged between 20-24 years had delivered one or more children [12]. In the fuzzy population model, the average female marriage age is represented by the AGE input factor.

6. Population Fuzzy Model

In section 5 emphasis has been placed on the four major factors that affect the population growth rate. The manner by which the female's income, education, extent of employment and marriage age influence the fertility has also been described. A linguistic description of the relationship between the noted factors and women's fertility using words such as the ones employed in every day language has, in addition, been shown possible. Well, such a description, which could be translated in the form of fuzzy inference rules, may turn out to be preferable over the use of direct mathematical relationships. In fact, this is what the investigators believe in due to the difficulty and complexity of modeling human behavior by precise mathematics [3, 13]. The belief is also supported by the approximate nature and humanistic aspects related to fuzzy inference modeling and the resulting simplicity that could be obtained while preserving the relevance of the model [19, 20]. In addition, the choice of this fuzzy inference methodology is motivated by the fact that for most practical purposes fairly accurate population growth figures as the ones obtained in this study could be found satisfactory and that a 100% accuracy cannot be achieved even by the most sophisticated available techniques.

As a result, fuzzy conditional inference statements like the ones expressed in equation (1), (7-9) apply. Hence, a fuzzy logic model can now be set and implemented to assess the behavior of the growth rate as determined by the stated factors. It is anticipated that such a methodology would work well in regions where no drastic population growth changes occur over short durations as is usually the case where no natural disasters or wars take place. As noted previously, the assignment of the membership functions ranges, ranges of the fuzzy model variables, and inference rules, is based on universally accepted standards and the interrelationship between the population growth factors (section 5). Also, tuning has been applied to the rules and membership functions using a developed CAD tool [4] for the simulation of fuzzy models and data for several countries [12, 17] related to values assumed by the growth influencing factors and the resulting growth.

By allowing each of the input and output variables as well as each of the input and output fuzzy sets that are assigned over these variables to occupy a range of values consistent with collected data while still respecting universal standards, we were able to partition each input variable namely, women's average monthly income, percentage of educated women, percentage participation of women in social activities and average marriage age into three fuzzy sets: Low, medium and high which are abbreviated LOW, MED and HI. These are shown in Figures 1-4. The output variable (population growth rate) is divided into five fuzzy sets: Very low, low, medium, high and very high (VLOW, LOW, MED, HI, and VHI) as in Figure 5. Thus, all combinations among the input fuzzy sets are considered and 81 inference rules (refer to Appendix) describing the fuzzy inference model are obtained.

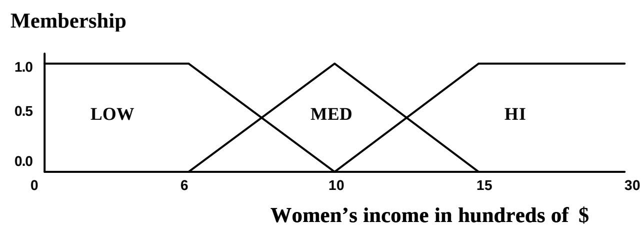

Figure 1. Input membership functions for women's income.

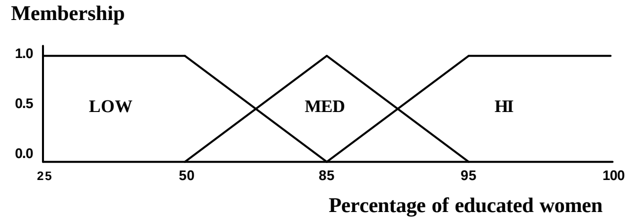

Figure 2. Input membership functions for % of educated women.

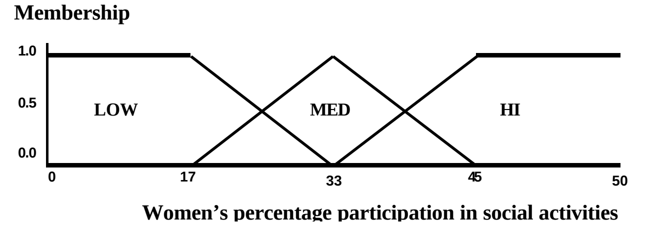

Figure 3. Input membership functions for % women's participation in social activities.

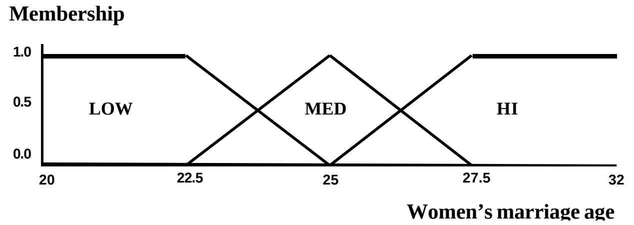

Figure 4. Input membership functions for women's marriage age.

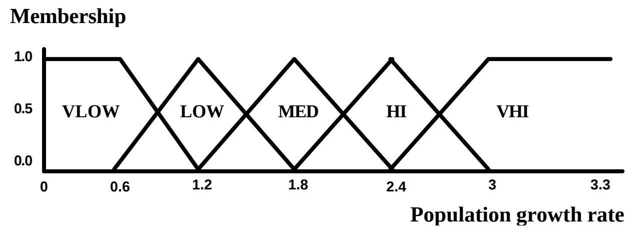

Figure 5. Output member functions for population growth rate.

7. Simulation Results

In order to obtain the crisp population growth rate for each specific input vector consisting of four crisp input values and, thus, apply the fuzzy population model to specific countries and compare with the actual historically registered results, a CAD tool for the simulation of fuzzy systems that has been developed by the authors in [4] is used. The tool permits the application of equation (4) to obtain the fuzzy output and the implementation of the different defuzzification strategies (FOM, COG and COS). The tool also permits the implementation of the defuzzification techniques which apply from within the rules without passing by the fuzzy output (Min-Max WAF and PAF). The CAD tool is functionally divided into two parts:

1. The fuzzy application which is responsible for managing files, editing, compiling, running the simulation and displaying the results in the form of two- or three-dimensional graphs.
2. The fuzzy printout utility which is used to print the graphs.

Data collection is obtained from studies carried out by the United Nations [12] and the World Bank [17]. The data available in these references on the four factors that affect the population growth rate in four different countries (Algeria, Austria, Chad, and France) are listed in Table 1. These data are used in the fuzzy population model to obtain the simulation results and compare with the actual population growth rates which are also listed in Table 1.

The simulation results, obtained in the form of population growth rates for the countries and years listed in Table 1 and using the different defuzzification strategies, are provided in Table 2. Four case studies where carried out for each country. Each case involved the determination of five population growth rate estimates, corresponding to the application of the five defuzzification schemes (COG, FOM, COS, WAF, and PAF) presented in section 3. The percentage errors of the simulation results which are obtained by comparison with the actual population growth rates are also listed in Table 2. The percentage error is calculated as follows:

$$\% \textit{ Error} = 100 \times \textit{Absolute} \; [(P_{act} - P_{sim}) / P_{act}] \qquad (10)$$

where P_{act} and P_{sim} are respectively the actual and simulated population growth rates.

As seen in Table 2, the percentage error varies with the applied defuzzification strategy. It also varies with the country and year. The variation of the percentage error with the defuzzification strategy is natural since different defuzzification methods lead to different crisp output values for the same input. The variation of the error with the considered country under the same defuzzification method could be thought of as related to the assignment of the input and output membership functions of the fuzzy model (i. e., shape and ranges), the inferences and the compatibility of this assignment with the particular situation in a specific country. That is, what is considered low income to some degree in one country may not be considered low to the same degree in another. Furthermore, the manner by which a low income affects population growth in one country may differ slightly from the manner a low income affects the growth in another country. This problem that can be dealt with by adjusting the membership functions and rules to provide reasonable results for the country of interest. In such a case, however, the fuzzy model generality would diminish.

A significant variation of the error with the year for the same country and same defuzzification strategy could result from some problems related to the validity of the applied defuzzification method and the corresponding fuzzy logic operations (sections 2 and 3). If we take for instance the results obtained for Austria in the years 1988 and 1989, then a significant error variation is obtained under all the defuzzification strategies except for the PAF. From this perspective, the PAF is the method that provides the best results compared to the other defuzzification strategies. As can be seen from Table 2, the 16 case studies carried out for the four countries when using the PAF defuzzification strategy provided errors within 5.2% which is an acceptable accuracy considering the source of errors in the simulation process as mentioned before. In fact, when averaging the error for the 16 simulation results obtained using the PAF method, a percentage error of 2.38 is obtained.

Table 1. Actual data for Algeria, Austria, Chad, and France.

Country	Year	Population Growth Rate (%)	Percentage of Educated Females (%)	Female Participation in Social Activities (%)	Female Income per Month (100 $)	Female Average Marriage Age (Years)
Algeria	1988	2.90%	53.00%	5.00%	$ 1.97	21.00
	1989	2.90%	55.00%	5.00%	$ 1.85	21.00
	1990	2.80%	57.00%	5.00%	$ 1.72	21.40
	1992	2.70%	57.00%	10.00%	$ 1.53	22.26
Austria	1988	0.75%	97.00%	37.00%	$ 12.89	27.70
	1989	0.70%	97.00%	38.00%	$ 14.42	27.70
	1990	0.65%	97.00%	39.00%	$ 15.88	27.70
	1992	0.60%	97.00%	40.00%	$ 18.42	27.70
Chad	1988	2.40%	25.00%	21.00%	$ 0.13	25.00
	1989	2.45%	25.00%	21.00%	$ 0.16	24.60
	1990	2.45%	30.00%	21.00%	$ 0.16	24.00
	1992	2.50%	30.00%	21.00%	$ 0.18	23.40
France	1988	0.70%	97.00%	37.00%	$ 13.41	28.02
	1989	0.70%	97.00%	38.00%	$ 14.85	28.02
	1990	0.60%	97.00%	39.00%	$ 16.24	28.02
	1992	0.60%	97.00%	40.00%	$ 18.60	28.02

For more illustration on the simulation results of the fuzzy population growth model, and on the advantage of the PAF method over the other defuzzification strategies, Figures 6-9 are provided. These figures show two dimensional plots of the obtained population growth rates versus marriage age (Figures 6 and 7) and versus income (Figures 8 and 9) using the five different defuzzification strategies. In Figures 6 and 7, the EDUC, PART, and INCOME input variables are fixed to the values in Table 1 for the year 1992 for the corresponding country. In Figures 8 and 9, the EDUC, PART and AGE are fixed to the values in Table 1 also for the year 1992 and the corresponding country. Figures 6-9 clearly indicate that the PAF technique provides relatively reasonable results that are free from abrupt transitions whereby the population growth output decreases smoothly as the age or income increases.

Reference can be made here to section 3 and [10] about the manner by which the PAF method was structured to use appropriate logic operations so as to account for the contribution of all competing rules and all fuzzy sets in the antecedent part of each rule. Now, with this observation of the results provided by the PAF method, the problem that was attributed to the possible lack of the same degree of compatibility between the membership functions and rules, on one hand, and the various countries, on the other hand, is no more of high significance and the generality of the model can be preserved. Yet, despite these observations, better rules should be sought to improve over the PAF results.

8. Conclusion

Fuzzy logic techniques, which have been devised and applied in the modeling and simulation of different types of complex humanistic processes, have been used in this study to introduce a fuzzy inference model for population growth. Due to the complexity of the classical population growth projection methods, which are based on demographic models and stochastic approaches, and the fact that fuzziness can naturally be attached to the factors that mainly affect the growth in population, the fuzzy model has been shown to fit well the population growth.

Table 2. Simulation results obtained from the fuzzy population model using the data in Table 1 and different defuzzification strategies (Actual: Actual population growth, G: Growth, E: Error).

Country			COG		FOM		COS		WAF		PAF	
	Year	Actual	%G	%E	%G	%E	%G	%E	%G	%E	%G	%E
Algeria	1988	2.90%	2.40%	17.24%	2.45%	15.51%	2.76%	4.82%	2.40%	17.24%	2.77%	4.48%
	1989	2.90%	2.40%	17.21%	2.49%	14.13%	2.76%	4.82%	2.40%	17.24%	2.75%	5.17%
	1990	2.80%	2.40%	14.28%	2.52%	10.00%	2.76%	1.42%	2.40%	14.28%	2.74%	2.14%
	1992	2.70%	2.40%	11.11%	2.52%	6.66%	2.76%	2.22%	2.40%	11.11%	2.70%	0.00%
Austria	1988	0.75%	0.77%	2.66%	0.85%	13.33%	0.88%	17.33%	0.74%	1.33%	0.76%	1.33%
	1989	0.70%	0.60%	14.28%	0.85%	21.42%	0.69%	1.42%	0.56%	20.00%	0.69%	1.42%
	1990	0.65%	0.53%	18.46%	0.90%	38.46%	0.61%	6.15%	0.47%	27.69%	0.64%	1.53%
	1992	0.60%	0.52%	13.33%	0.85%	41.66%	0.60%	0.00%	0.47%	21.66%	0.60%	0.00%
Chad	1988	2.40%	2.40%	0.00%	2.58%	7.5%	2.76%	15.00%	2.40%	0.00%	2.45%	2.08%
	1989	2.45%	2.40%	2.04%	2.62%	6.93%	2.76%	12.65%	2.40%	2.04%	2.48%	1.22%
	1990	2.45%	2.40%	2.04%	2.61%	6.53%	2.76%	12.65%	2.40%	2.04%	2.55%	4.08%
	1992	2.50%	2.40%	4.00%	2.55%	2.00%	2.76%	10.40%	2.40%	4.00%	2.60%	4.00%
France	1988	0.70%	0.77%	10.00%	0.90%	28.57%	0.81%	15.71%	0.71%	1.42%	0.72%	2.85%
	1989	0.70%	0.52%	25.71%	0.85%	21.42%	0.60%	14.28%	0.47%	32.85%	0.68%	2.85%
	1990	0.60%	0.53%	11.66%	0.90%	50.00%	0.61%	1.66%	0.47%	21.66%	0.63%	5.00%
	1992	0.60%	0.52%	13.33%	0.85%	41.66%	0.60%	0.00%	0.47%	21.66%	0.60%	0.00%

What is needed to estimate or even predict the population growth rate, in some specific country or region over a short period of time, one year for example, is to have some rough crisp values of the input factors that are used as the input variables of the fuzzy population model. These values can be easily obtained by examining samples from the population of the country or region at the beginning of the year of concern, the available data in previous years and some governmental programs that are expected to affect the values assumed by the input variables of the fuzzy population model. In any case, highly precise input values are not needed since the fuzzy model allows for some imprecision without having major effects on the output (see Figures 6-9 case (e)).

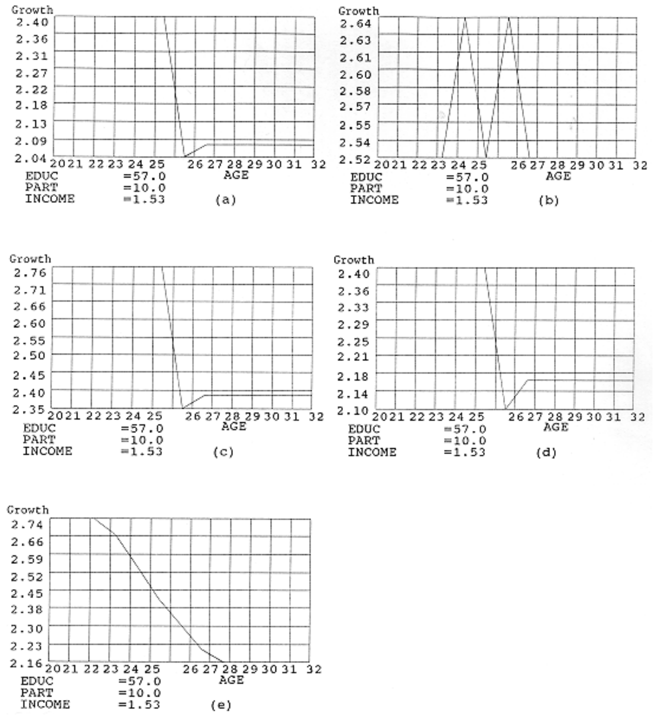

Figure 6. Population growth rate versus female marriage age for fixed EDUC, PART, and INCOME input variables (Algeria 1992). (a) COG, (b) FOM, (c) COS, (d) WAF, (e) PAF.

Moreover, for the fuzzy model to provide the best results in the determination of population growth in a specific country, it is advisable that the membership functions (number, shapes and ranges), the inference rules and the defuzzification method be selected so that the obtained results match the historically recorded population growth rates with the smallest possible percentage error. As seen in the results offered in this study (section 7, Table 2, and Figures 6-9) the PAF defuzzification strategy provided the growth rates closest to the actual ones. So, it is advisable to use this strategy. Yet, an improvement of the accuracy of the results through the additional adjustment and tuning the rules and membership functions, is still recommendable.

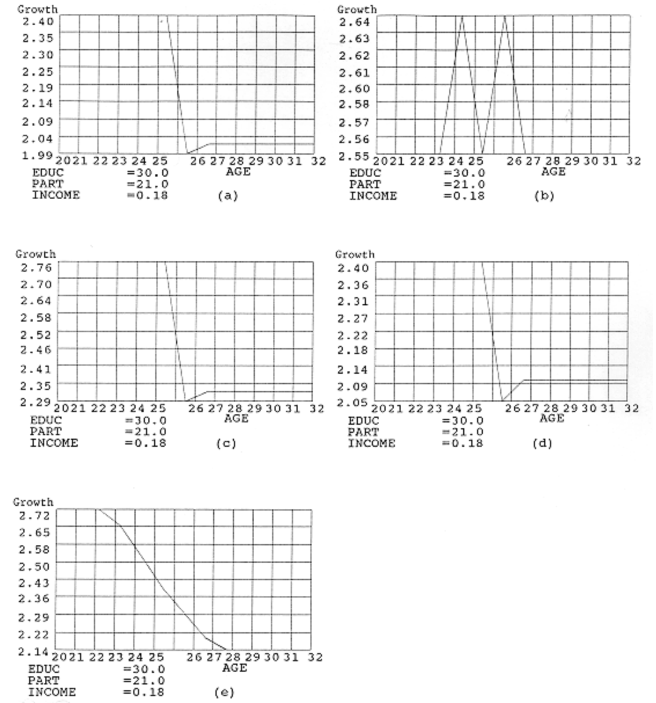

Figure 7. Population growth rate versus female marriage age for fixed EDUC, PART, and INCOME input variables (Chad 1992). (a) COG, (b) FOM, (c) COS, (d) WAF, (e) PAF.

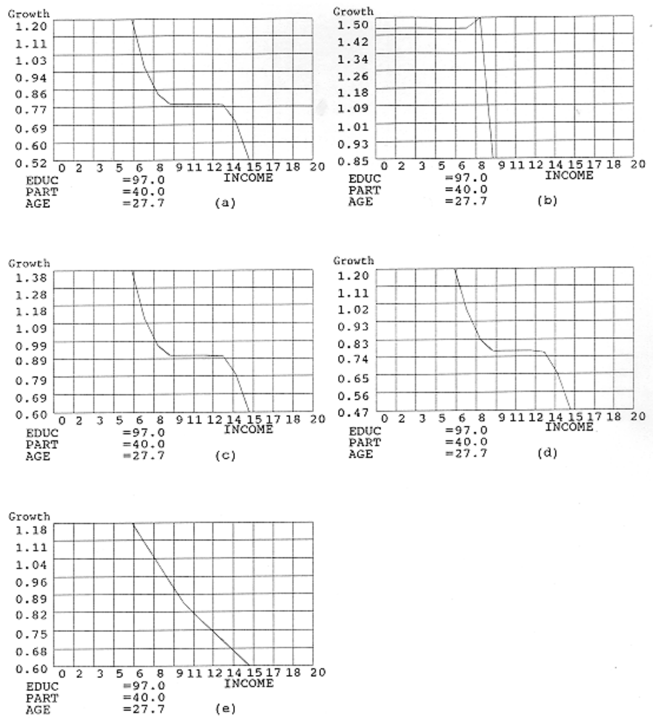

Figure 8. Population growth rate versus female's monthly income for fixed EDUC, PART, and AGE input variables (Austria 1992). (a) COG, (b) FOM, (c) COS, (d) WAF, (e) PAF.

We note here that the mortality as well as migration rates have been assumed constant so that the change in population growth rate from one year to another is only influenced by the fertility rate. Even if the mortality and migration rates are subject to small changes, they would not have a great deal of effect on the provided model and its results. Since the fuzzy model was designed based on the use of data that accounts for the net result of fertility, mortality and migration represented by population growth and it is tolerant to imprecision. The fuzzy model, however, would not work properly when a sudden considerable change in mortality rate, say, takes place (e. g., natural disaster) since in such a situation the growth rate would decrease sharply from one year to another as compared to its normal behavior assuming that the fertility rate remains normal. This sharp decrease can not be accommodated by the fuzzy model due to the smoothness of its input-output characteristics (see Figures 6-9 case (e)).

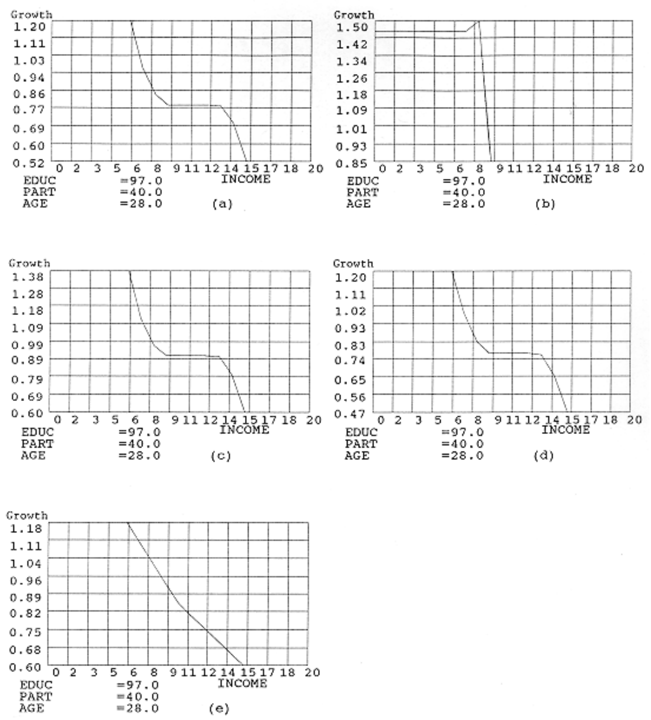

Figure 9. Population growth rate versus female's monthly income for fixed EDUC, PART, and AGE input variables (France 1992). (a) COG, (b) FOM, (c) COS, (d) WAF, (e) PAF.

Further research in the area should deal with the assignment of membership functions, rules and defuzzification strategies and the application of fuzzy adaptive techniques in order to even improve the obtained simulation results. The use of fuzzy adaptive techniques should rely on available data (such as Table 1), and data obtained through simulation (Table 2) so as the membership functions, inference rules and defuzzification method are automatically adjusted and the error is brought to the lowest possible level. A topic that also deserves further investigation consists of dealing with the issues that arise when the population growth prediction is desired over a long period of time. In such a case, long-range predicted values for the inputs of the fuzzy population model are needed. These predicted values could be obtained by applying fuzzy predictive modeling to the input variables themselves and the factors affecting these variables. Another approach is to modify the presented fuzzy model so as to make it predictive through the use of the rate of change in EDUC, INCOME, PART, and AGE as input variables and change in population growth as output variable. The design and adjustment of the new fuzzy model then would have to rely on past and present values of the noted variables and the differences observed over the years so as to assess the future population growth.

It is worth noting here finally that no attempts have been made in this study to compare the available classical fertility models with the offered fuzzy model, which is mainly concerned, as of now, with the present rather than the future figures. The reason is that the fuzzy and classical models are derived based on different setting and configurations as to the influencing factors and objectives. The original intention behind the development of classical fertility and other demographic models was to draw conclusions related to their direct causes; such as abortion, contraceptive use, etc. [3]. From these conclusions methods for controlling fertility, mortality and migration may become easier to perceive. Classical population projection methods came about as a secondary added value to the demographic models. The objective of this study, however, is to provide modeling for the population growth directly. Secondary objectives could be related to figuring out methods to influence women's education, employment conditions, and income so as to control population growth. Also, the classical models have been reported to lead to conflicting results in some instances. Comparison of the fuzzy model results with actual data is good in itself and determines the quality of a demographic model and whether it can be used for empirical research [3]. After all, this study can be considered as a good attempt aimed at laying the foundation of and triggering further research in the area of fuzzy modeling as it relates to demographic issues.

Appendix: Inference Rules

The following are the inference rules implemented for the fuzzy population model:

IF INCOME IS LOW AND EDUC IS LOW AND PART IS LOW AND AGE IS LOW THEN POP IS VHI
IF INCOME IS LOW AND EDUC IS LOW AND PART IS LOW AND AGE IS MED THEN POP IS VHI
IF INCOME IS LOW AND EDUC IS LOW AND PART IS LOW AND AGE IS HI THEN POP IS HI
IF INCOME IS LOW AND EDUC IS LOW AND PART IS MED AND AGE IS LOW THEN POP IS VHI

IF INCOME IS LOW AND EDUC IS LOW AND PART IS MED AND AGE IS MED THEN POP IS HI
IF INCOME IS LOW AND EDUC IS LOW AND PART IS MED AND AGE IS HI THEN POP IS MED
IF INCOME IS LOW AND EDUC IS LOW AND PART IS HI AND AGE IS LOW THEN POP IS HI
IF INCOME IS LOW AND EDUC IS LOW AND PART IS HI AND AGE IS MED THEN POP IS MED
IF INCOME IS LOW AND EDUC IS LOW AND PART IS HI AND AGE IS HI THEN POP IS MED
IF INCOME IS LOW AND EDUC IS MED AND PART IS LOW AND AGE IS LOW THEN POP IS VHI
IF INCOME IS LOW AND EDUC IS MED AND PART IS LOW AND AGE IS MED THEN POP IS HI
IF INCOME IS LOW AND EDUC IS MED AND PART IS LOW AND AGE IS HI THEN POP IS MED
IF INCOME IS LOW AND EDUC IS MED AND PART IS MED AND AGE IS LOW THEN POP IS HI
IF INCOME IS LOW AND EDUC IS MED AND PART IS MED AND AGE IS MED THEN POP IS MED
IF INCOME IS LOW AND EDUC IS MED AND PART IS MED AND AGE IS HI THEN POP IS MED
IF INCOME IS LOW AND EDUC IS MED AND PART IS HI AND AGE IS LOW THEN POP IS MED
IF INCOME IS LOW AND EDUC IS MED AND PART IS HI AND AGE IS MED THEN POP IS MED
IF INCOME IS LOW AND EDUC IS MED AND PART IS HI AND AGE IS HI THEN POP IS MED
IF INCOME IS LOW AND EDUC IS HI AND PART IS LOW AND AGE IS LOW THEN POP IS HI
IF INCOME IS LOW AND EDUC IS HI AND PART IS LOW AND AGE IS MED THEN POP IS MED
IF INCOME IS LOW AND EDUC IS HI AND PART IS LOW AND AGE IS HI THEN POP IS MED
IF INCOME IS LOW AND EDUC IS HI AND PART IS MED AND AGE IS LOW THEN POP IS MED
IF INCOME IS LOW AND EDUC IS HI AND PART IS MED AND AGE IS MED THEN POP IS MED
IF INCOME IS LOW AND EDUC IS HI AND PART IS MED AND AGE IS HI THEN POP IS MED
IF INCOME IS LOW AND EDUC IS HI AND PART IS HI AND AGE IS LOW THEN POP IS MED
IF INCOME IS LOW AND EDUC IS HI AND PART IS HI AND AGE IS MED THEN POP IS MED
IF INCOME IS LOW AND EDUC IS HI AND PART IS HI AND AGE IS HI THEN POP IS LOW
IF INCOME IS MED AND EDUC IS LOW AND PART IS LOW AND AGE IS LOW THEN POP IS VHI
IF INCOME IS MED AND EDUC IS LOW AND PART IS LOW AND AGE IS MED THEN POP IS HI
IF INCOME IS MED AND EDUC IS LOW AND PART IS LOW AND AGE IS HI THEN POP IS MED
IF INCOME IS MED AND EDUC IS LOW AND PART IS MED AND AGE IS LOW THEN POP IS HI
IF INCOME IS MED AND EDUC IS LOW AND PART IS MED AND AGE IS MED THEN POP IS MED
IF INCOME IS MED AND EDUC IS LOW AND PART IS MED AND AGE IS HI THEN POP IS MED
IF INCOME IS MED AND EDUC IS LOW AND PART IS HI AND AGE IS LOW THEN POP IS MED
IF INCOME IS MED AND EDUC IS LOW AND PART IS HI AND AGE IS MED THEN POP IS MED
IF INCOME IS MED AND EDUC IS LOW AND PART IS HI AND AGE IS HI THEN POP IS MED
IF INCOME IS MED AND EDUC IS MED AND PART IS LOW AND AGE IS LOW THEN POP IS HI
IF INCOME IS MED AND EDUC IS MED AND PART IS LOW AND AGE IS MED THEN POP IS MED
IF INCOME IS MED AND EDUC IS MED AND PART IS LOW AND AGE IS HI THEN POP IS MED
IF INCOME IS MED AND EDUC IS MED AND PART IS MED AND AGE IS LOW THEN POP IS MED
IF INCOME IS MED AND EDUC IS MED AND PART IS MED AND AGE IS MED THEN POP IS MED
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IF INCOME IS HI AND EDUC IS LOW AND PART IS LOW AND AGE IS LOW THEN POP IS HI
IF INCOME IS HI AND EDUC IS LOW AND PART IS LOW AND AGE IS MED THEN POP IS MED
IF INCOME IS HI AND EDUC IS LOW AND PART IS LOW AND AGE IS HI THEN POP IS MED
IF INCOME IS HI AND EDUC IS LOW AND PART IS MED AND AGE IS LOW THEN POP IS MED
IF INCOME IS HI AND EDUC IS LOW AND PART IS MED AND AGE IS MED THEN POP IS MED
IF INCOME IS HI AND EDUC IS LOW AND PART IS MED AND AGE IS HI THEN POP IS MED
IF INCOME IS HI AND EDUC IS LOW AND PART IS HI AND AGE IS LOW THEN POP IS MED
IF INCOME IS HI AND EDUC IS LOW AND PART IS HI AND AGE IS MED THEN POP IS MED
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IF INCOME IS HI AND EDUC IS MED AND PART IS LOW AND AGE IS HI THEN POP IS MED
IF INCOME IS HI AND EDUC IS MED AND PART IS MED AND AGE IS LOW THEN POP IS MED
IF INCOME IS HI AND EDUC IS MED AND PART IS MED AND AGE IS MED THEN POP IS MED

IF INCOME IS HI AND EDUC IS MED AND PART IS MED AND AGE IS HI THEN POP IS LOW
IF INCOME IS HI AND EDUC IS MED AND PART IS HI AND AGE IS LOW THEN POP IS MED
IF INCOME IS HI AND EDUC IS MED AND PART IS HI AND AGE IS MED THEN POP IS LOW
IF INCOME IS HI AND EDUC IS MED AND PART IS HI AND AGE IS HI THEN POP IS VLOW
IF INCOME IS HI AND EDUC IS HI AND PART IS LOW AND AGE IS LOW THEN POP IS MED
IF INCOME IS HI AND EDUC IS HI AND PART IS LOW AND AGE IS MED THEN POP IS MED
IF INCOME IS HI AND EDUC IS HI AND PART IS LOW AND AGE IS HI THEN POP IS LOW
IF INCOME IS HI AND EDUC IS HI AND PART IS MED AND AGE IS LOW THEN POP IS MED
IF INCOME IS HI AND EDUC IS HI AND PART IS MED AND AGE IS MED THEN POP IS LOW
IF INCOME IS HI AND EDUC IS HI AND PART IS MED AND AGE IS HI THEN POP IS VLOW
IF INCOME IS HI AND EDUC IS HI AND PART IS HI AND AGE IS LOW THEN POP IS LOW
IF INCOME IS HI AND EDUC IS HI AND PART IS HI AND AGE IS MED THEN POP IS VLOW
IF INCOME IS HI AND EDUC IS HI AND PART IS HI AND AGE IS HI THEN POP IS VLOW

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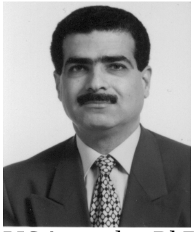

Jean Saade received the BSc degree in Physics from the Lebanese University, Lebanon, in 1979 with a scholarship to pursue graduate studies. He then received the MSc degree in electrical engineering in 1982 from Boston University, MA, USA, and a PhD degree, also in electrical engineering, from Syracuse University, NY, USA, in 1987. He taught for four years at Syracuse University during his PhD studies and served for one year as a visiting assistant professor at the same University after graduating. In 1988, he joined the American University of Beirut, Lebanon, and is currently a professor at the Department of Electrical and Computer Engineering. His recent publications have been concerned with the development of fuzzy controllers modeling algorithms and learning schemes based on expert data. This is in addition to the development of novel defuzzification methods for fuzzy controllers. He has also received special invitation to present his research at high-level international conferences and VIP forums. He is a member of the International Fuzzy Systems Association (IFSA) and the European Society for Fuzzy Logic and Technology (EUSFLAT).